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BY

EDWARD L. NICHOLS,
ERNEST MERRITT, and FREDERICK BEDELL

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MAY, 1912

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By EDWARD L. NICHOLS

ERNEST MERRITT, AND FREDERICK BEDELL

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THE

PHYSICAL REVIEW.

THE THEORETICAL AND EXPERIMENTAL DETERMINATION OF REFLECTION COEFFICIENTS.

By J. T. Tate.

THE optical constants of absorbing media have been studied by two general methods, according as the observations have been made on transmitted or on reflected light. The transmission determinations comprise notably Kundt's[1] metallic prism method for refractive indices and Wernicke's[2] direct photometric method for absorption coefficients. Owing, however, to the large experimental error introduced in measuring the thickness of the necessarily very thin films used, the "reflection" method, which obviates this, has in recent years been applied almost exclusively. The "reflection" method consists in a polarimetric analysis of the ellipticity and rotation produced in plane polarized light by reflection from the surface of the medium under investigation. Drude,[3] who was the first to apply this to an extensive study of the optical constants of absorbing media, has conclusively shown from a consideration of the experimental errors involved that the values obtained by polarimetric measurements are decidedly more accurate than those by any other known method.

It must be noted, however, that the values for the optical constants obtained by these two entirely different methods (the reflection and the transmission) have not in general agreed within the errors of observation and that an explanation of this variation must take into account the essential difference between them. The one deals with the light which has been affected by the surface of the body only, while the other deals with that which has been affected by the medium as a whole. The discrepancies in the observed magnitudes indicate, therefore, that the media

[1] Kundt, A., Wied. Ann., 34, p. 469, 1888.
[2] Wernicke, W., Pogg. Ann. Ergbd., 8, p. 65, 1878.
[3] Drude, P., Ann. d. Physik, 39, p. 481, 1890.

dealt with are not strictly speaking homogeneous but possess characteristics at their surfaces differing from those within.

This condition of affairs has been emphasized in the more recent investigations by the reflection method. Much uncertainty as to the reliability of the results obtained by it has arisen on account of the inability of different observers, and even of the same observer, to obtain consistent results from any one substance, it being found that changing the method of polishing, or even repolishing by the same method, changes decidedly the characteristics of the surface. Drude[1] first, and more recently Minor[2] and Tool,[3] have investigated this fact and from their results the following conclusions may be drawn:

1. The mere process of polishing modifies the properties of the surface considerably, increasing the absorption and reflection coefficients and decreasing the refractive indices. Drude[1] showed this conclusively by making observations on surfaces of cleavage of lead sulphid first in their natural condition, then after polishing. From the polished surface he obtained an absoprtion coefficient 35 per cent. larger, and a refractive index proportionately smaller, than the same surface gave in its natural condition.

2. Impurities worked into the surface during the polishing process augment the above changes.

3. An incomplete polish or matt surface usually tends to decrease the values of the constants.

4. Films formed on the surface by oxidation or other chemical change ordinarily have the same effect as a matt surface.

5. With but few exceptions, however, the *form* of the curves is not affected by the polishing process or by impurities.

6. The efforts which have been made by the different observers to obtain a so-called "standard" method of preparing the surfaces have met with only doubtful success on account of the very large effect produced by even very slight changes in the method of polishing.

From the above conclusions it is evident that the reflection method does not give true values for the optical constants of the *body* of the medium but only those which are characteristic of the particular *surface* under investigation. Nothing is to be gained from a comparison of the results obtained by different observers except the *form* of the dispersion curves and rough average values for the constants. Despite these facts, the reflection method, with any one particular surface, does give con-

[1] Drude, P., Ann. d. Physik, 36, p. 532, 1889.
[2] Minor, R. S., Ann. d. Physik, 10, p. 581, 1903.
[3] Tool, A. Q., Phys. Rev., 31, p. 1, 1910.

sistent results for polarimetric measurements taken at varying angles of incidence. It is important therefore to determine whether all of the constants obtainable from any one surface by reflection measurements are consistent with each other according to the accepted theories.

The object of the present investigation was to compare the reflection coefficients computed from the optical constants obtained polarimetrically with the reflection coefficients actually observed on the same surface. The comparisons which have been made by Drude,[1] Minor,[2] Tool,[3] and others between the theoretical and observed magnitudes of the reflection coefficients are all open to the same weakness in that the theoretical values were calculated for the particular mirror which happened to be under investigation and compared with those which had been experimentally determined by other observers on other mirrors. From the facts brought out above it is evident that any close agreement would be largely accidental and hence of little or no value as a rigid test of the theory. As might be expected they found the agreement to be far from satisfactory in almost every case. On the other hand the comparison made in the present investigation, based on observations taken on one and the same surface and therefore free from the above weakness, shows excellent agreement for all surfaces tested.

Apparatus.

For Polarimetric Measurements.—To measure the ellipticity and rotation produced in plane polarized light by reflection from the surface of the absorbing medium the analyzing system devised by Tool was used, in which, for the sake of accuracy, the Brace Half Shade is introduced as a modification of the well-known Stokes' analyzer. With sunlight (analyzed by a spectral system) this instrument is capable of measuring an ellipticity of approximately unity to .05 per cent. and of 0.2 to 1 per cent., while the rotation is accurately obtained to about .05 per cent.

For Photometric Measurements.—A Brace spectro-photometer, in connection with a special reflecting device, was used for the photometric measurements. With this instrument an accuracy in the reflection coefficients of about .5 per cent. (under best conditions) to 1 per cent.—depending on the color and the surface dealt with—was obtained. An acetylene jet which gave a steady flame and whose illumination was uniform over a sufficient area served as a light source. Light from a very small area on one side of the flame was reflected into the collimator C_1

[1] Drude, P., loc. cit., 1 preceding page.
[2] Minor, R. S., loc. cit.
[3] Tool, A. Q., loc. cit., 4.

(see Fig. 1) by the total reflecting prisms at P. Light from the same part of the flame on the other side was directed through the reflecting instrument, finally entering the other collimator C_2. To obtain the relative intensities of reflected and unreflected light it was then only necessary to direct the light into the collimator C_2 successively with and without reflection at the mirror surface, and measure the intensity in each case with the light entering the collimator C_1 as a standard of comparison. It is of course essential that the optical path and the equivalent focus of the light source be the same for both unreflected and reflected rays. The reflection instrument (shown diagrammatically in vertical section in Fig. 2) was designed to secure these conditions. The light enters the instrument through the fixed total reflecting prism A, so adjusted as to reflect it up along the axis of rotation of an arm which carries other total reflecting prisms B, C and D. After reflection at their surfaces (as indicated in the figure) the light finally reaches the mirror M whose surface lies in the axis of rotation of the system of prisms. By M it is reflected directly into the collimator C_2. With the mirror M removed and the prisms rotated into the position indicated by the broken lines, the same light which was previously reflected by M into the collimator, now passes directly into the collimator. It is readily seen that the optical path of the ray in both cases is the same and, provided the mirror M is perfectly plane, the equivalent focus of the light source is also the same. With this instrument, therefore, it is possible to readily adjust for measuring the reflection coefficient at any angle of incidence insuring at the same time the identity of optical path.

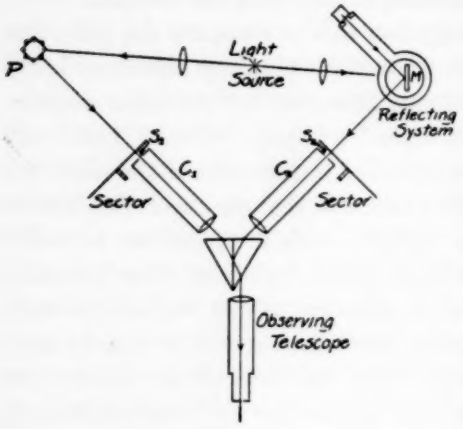

Fig. 1.

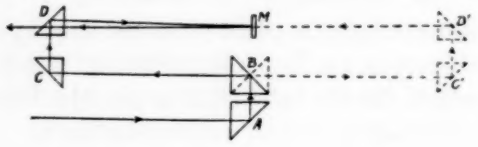

Fig. 2.

Comparisons of intensity were made in the usual way by means of the graduated sectors and the adjustable slits S_1 and S_2.

Formulæ.

For calculating the refractive indices and absorption coefficients the following customary equations were used.

$$m_\theta = -\frac{\sin\theta \operatorname{tg}\theta \cos 2\psi \cos 2\gamma}{1 - \cos 2\psi \sin 2\gamma},$$

$$m_\theta \kappa_\theta = \frac{\sin\theta \operatorname{tg}\theta \sin 2\psi}{1 - \cos 2\psi \sin 2\gamma}, \tag{1}$$

$$\kappa_\theta = -\frac{\operatorname{tg} 2\psi}{\cos 2\gamma},$$

in which m_θ is defined by the equation

$$m_\theta = \nu_\theta \cos\rho,$$

where ν_θ is the refractive index of the medium at the angle of refraction ρ and the corresponding angle of incidence θ. The quantity m_θ may therefore be defined as the ratio of the velocity of the incident light to the velocity of the refracted light measured along the normal to the planes of equal amplitude. The quantity 2ψ is a measure of the ellipticity of the reflected light defined by the relation

$$E = tg\psi,$$

where E is the ratio of the minor to the major axis of the ellipse. The quantity 2γ is the so-called azimuth of restored polarization of the elliptic vibration measured with the plane of polarization of the incident light as the plane of reference. The quantity κ_θ is the absorption coefficient of the medium for the angle of incidence θ such that $e^{-2\pi\kappa\theta}$ gives the ratio of the amplitudes of the vibration at two successive points of equal phase on any normal to the planes of equal amplitude. In all cases where the light enters from a transparent medium these planes of equal amplitude are parallel to the dividing surface.

These quantities, m_θ, $m_\theta\kappa_\theta$ and κ_θ, which are strictly speaking properties of the medium only for the particular angle of incidence θ, are related to the corresponding constants of the medium at normal incidence by the following equations:

$$\begin{aligned} 2\nu_0^2 &= \sqrt{(m_\theta^2 - m_\theta^2\kappa_\theta^2 + \sin^2\theta)^2 + 4m_\theta^4\kappa_\theta^2} + (m_\theta^2 - m_\theta^2\kappa_\theta^2 + \sin^2\theta), \\ 2\nu_0^2\kappa_0^2 &= \sqrt{(m_\theta^2 - m_\theta^2\kappa_\theta^2 + \sin^2\theta)^2 + 4m_\theta^4\kappa_\theta^2} - (m_\theta^2 - m_\theta^2\kappa_\theta^2 + \sin^2\theta), \end{aligned} \tag{2}$$

where ν_0 and κ_0 are respectively the refractive index and absorption coefficient at normal incidence. For convenience in computation where $\sin^2\theta$ is small in comparison with ν_0^2 and κ_0^2 these equations have been expanded in series involving powers of $\sin^2\theta$ as follows:

$$\nu_0 = m_\theta \left[1 + \frac{\sin^2\theta}{2m_\theta^2(1+\kappa_\theta^2)} + \frac{1}{2}\left(\frac{m_\theta^2\kappa_\theta^2}{m_\theta^2(1+\kappa_\theta^2)} - \frac{1}{4}\right)\frac{\sin^4\theta}{m^4(1+\kappa_\theta^2)^2} - \cdots \right],$$

$$\nu_0\kappa_0 = m_\theta\kappa_\theta \left[1 - \frac{\sin^2\theta}{2m_\theta^2(1+\kappa_\theta^2)} + \frac{1}{2}\left(\frac{m_\theta^2}{m_\theta^2(1+\kappa_\theta^2)} - \frac{1}{4}\right)\frac{\sin^4\theta}{m_\theta^4(1+\kappa_\theta^2)^2} - \cdots \right]. \quad (3)$$

From ν_0 and κ_0 the reflection coefficient at normal incidence J_0 is given by the relation

$$J_0 = \frac{\nu_0^2(1+\kappa_0^2) + 1 - 2\nu_0}{\nu_0^2(1+\kappa_0^2) + 1 + 2\nu_0}.$$

From the high accuracy possible in measuring the ellipticity and rotation, it is interesting to note that the probable error in J_0 as calcuated from the polarimetric measurements was found to be only about .1 per cent. while that in the photometrically determined value was from .5 to 1 per cent. If the polarimetrically determined values of J_0 are reliable it must therefore be concluded that the values thus obtained are much more accurate than those obtainable by any direct photometric determination.

EXPERIMENTAL RESULTS.

Silver.—The silver was chemically deposited on a plate of glass perfectly free from double refraction, and observations made both on the air-silver and on the glass-silver surface. Unfortunately it was impossible to secure a naturally brilliant air-silver surface without recourse to polishing with chamois skin. For polarimetric measurements on the glass-silver surface a 60° prism of the same optical glass as the plate upon which the silver was deposited was cemented to it with Canada balsam (Fig. 3). This allowed the light to enter and leave the glass normal to its surface thus obviating any difficulties arising from rotation of the azimuth of the incident plane polarized light or of change in the angle of incidence by refraction. To obtain photometric measurements atnormal incidence on the same surface this prism was removed.

Fig. 3.

The results obtained from both surfaces are given in Tables I. and II. To obtain a correct value for the "observed" reflection from the silver-glass surface the theoretical coefficient (.04) was used for the glass-air boundary. By calculation this surface was found to produce an inappreci-

TABLE I.

Silver.

(Light incident in Air.)

Wave-length ($\mu\mu$).	n_0	κ_0	$n_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	.270	12.02	3.245	.911	.905
500	.273	13.03	3.557	.923	.927
540	.279	13.95	3.892	.933	.930
580	.284	14.77	4.195	.941	.948
620	.291	15.53	4.519	.947	.950
660	.299	16.28	4.868	.953	.952
700	.308	16.87	5.196	.957	.953

TABLE II.

Silver.

(Light incident in Glass.)

Wave-length ($\mu\mu$).	n_0	κ_0	$n_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	.339	6.90	2.340	.817	.810
500	.343	7.72	2.648	.844	.840
540	.348	8.51	2.962	.869	.862
580	.356	9.26	3.301	.889	.885
620	.365	9.93	3.625	.900	.902
660	.375	10.46	3.921	.913	.919
700	.386	10.96	4.230	.922	.930

able effect. A comparison of these tables shows a striking difference in the properties of the silver at the two surfaces. The absorption coefficients κ_0 on the side next the glass have about one half the magitude of those next the air. Similarly the calculated value of the refractive index n_0 of the silver on the side next the glass is nearly double that on the other. It is impossible to calculate this accurately because, while the actual refractive index of the glass was about 1.51, a polarimetric measurement of this constant on a part of the surface not covered by the silver gave a value of 1.30. Using, however, the refractive index as 1.51 that of the silver for a wave-length of 580$\mu\mu$ would be $1.51 \times .356 = .538$, while the value 1.30 gives .463. For the same wave-length on the air side a distinctly smaller value than either of these, namely, .284 was obtained. The ratio of both absorption coefficients and refractive indices (for the two silver surfaces) is about the same as that obtained by Drude[1] on lead sulphid before and after polishing. It is interesting to note here, as shown by Tool in a number of cases, that the product of refractive index and absorption coefficient is practically the same for both surfaces. Another interesting difference between the two surfaces is that relatively less blue light is reflected from the glass-silver surface than from the other.

[1] Drude, P., loc. cit., 4.

A comparison of the calculated and observed values for the reflection coefficients shows in general a very close agreement (see Fig. 4). That the small differences—the maximum being less than 1 per cent., the average less than .5 per cent.—arise from experimental errors is apparent from the fact that the observed values lie on both sides of the curve for the calculated values.

Gold and Copper.—The large variation in the visible spectrum shown by the reflection coefficients of gold and copper made these very desirable substances for the present investigation. As was stated in the paragraph on spectro-photometric adjustments it is essential that the mirror surfaces be very nearly plane and reasonably free from scratches. In attempting to surface these two metals great difficulty was experienced in realizing these requirements, owing to their softness and the consequent difficulties in securing a good polish. The following electrolytic process was finally adopted and proved to be a very satisfactory solution of the problem. A large mirror of speculum metal whose surface was plane and free from scratches was cut into small mirrors and upon these the copper and gold were deposited electrolytically. The resulting surfaces were exact duplicates of the speculum metal surface in respect to both planeness and polish—the plating solutions and current density being so chosen that perfect mirrors were produced without any subsequent polish being necessary. Although the individual mirrors obtained in this way were very satisfactory it was found impossible to duplicate, within one per cent., the results with different samples, even though the conditions of deposit were carefully reproduced.

The gold mirror was prepared by electrolytic deposition from a solution made up as follows. Sodium phosphate 5 gr., potassium hydrate 3 gr., potassium cyanide 15 gr., gold chloride 1 gr., water 1 liter. This solution was warmed to 50° C. and the current regulated to about .005 ampere per square centimeter.

The results (Table III., Fig. 4), compared with those obtained by other investigators on specimens which had been polished, again show quite strikingly the effect of the polishing process in changing the characteristics of the surface. The absorption coefficients of the untreated surface are from 20 to 50 per cent. smaller than those observed by Tool on one which had to be polished, while the refractive indices are correspondingly higher, so that the value of $\nu_0 K_0$ does not differ greatly from his.

As with silver, the agreement between calculated and observed values of the reflection coefficients is very close, the fluctuation being on both sides of the smooth curve—the maximum difference being 2.5 per cent. on the steepest slope of the curve, and the mean 1 per cent. The character

of this curve is slightly different from that obtained by Tool—relatively less red light being reflected, the mirror used here appearing more greenish-yellow than his.

TABLE III.

Gold.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.662	1.098	1.825	.362	.370
480	1.533	1.219	1.865	.381	.380
500	1.330	1.510	2.008	.438	.430
520	1.104	1.981	2.187	.521	.530
540	.937	2.717	2.545	.634	.618
560	.805	3.503	2.819	.703	.708
580	.729	4.144	3.023	.760	.758
600	.666	4.864	3.239	.800	.792
620	.645	5.294	3.414	.820	.812
640	.630	5.687	3.581	.837	.840
660	.636	5.862	3.727	.847	.852
680	.617	6.255	3.859	.859	.853

The copper mirror was deposited from a solution made up as follows: Copper acetate 25 gr., sodium carbonate 25 gr., sodium bisulphite 20 gr., potassium cyanide 50 gr., water 1 liter. The solution was used at room temperature and the current density regulated to .005 ampere per square centimeter. The same differences in the constants ν_0 and κ_0 (Table IV., Fig. 4) of the surface thus prepared as compared with those obtained on a polished surface by Tool is again noticed here—the magnitude and direction of the change being about the same as with gold. Expecially striking therefore is the close agreement between our values of $\nu_0\kappa_0$—the difference not exceeding 2 per cent., although the differences in ν_0 and κ_0 amount to about 40 per cent.

TABLE IV.

Copper.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.570	1.440	2.261	.463	.460
480	1.531	1.503	2.301	.477	.480
500	1.500	1.580	2.370	.494	.490
520	1.434	1.675	2.402	.510	.512
540	1.378	1.788	2.464	.530	.540
560	1.269	2.018	2.561	.566	.575
580	1.067	2.659	2.837	.653	.655
600	.980	3.250	3.185	.721	.710
640	.958	3.685	3.530	.764	.765
660	.996	3.716	3.701	.775	.777
700	1.035	3.688	3.817	.778	.786

The agreement between the observed and calculated values of the reflection coefficients is again very close, the largest variation for any one wave-length being about 2 per cent. and the mean 1 per cent. That these differences are also due to photometric errors is shown by the fact that the observed values fluctuate about equally on both sides of the curve of calculated values. The reflection coefficients for this surface are about 8 per cent. lower than those given by Tool but the form of the curve is the same.

Steel.—The steel mirror was one from a König tuning fork possessing a very perfect surface. It was thoroughly cleansed and rubbed with chamois skin just before the observations were made. The results (Table V., Fig. 4) agree very closely with those obtained by other observers on polished steel—differing from those given by Tool by less than 2 per cent.

TABLE V.

Steel.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.930	1.535	2.965	.556	.555
500	2.130	1.460	3.110	.562	.560
540	2.300	1.405	3.230	.569	.560
580	2.435	1.355	3.300	.571	.573
620	2.536	1.340	3.395	.578	.573
660	2.635	1.310	3.450	.581	.580
700	2.695	1.305	3.515	.587	.580

The calculated and observed reflection coefficients show close agreement—the fluctuation being on both sides of the curve and in no case exceeding 1.5 per cent. It is important to note one thing shown by all the reflection coefficient curves and most strikingly by the curve for steel, namely, that far more consistent and accurate results are evidently obtained by polarimetric measurements than by the more direct photometric method—the points representing the calculated values always lying closer to the smooth curve than the values directly observed.

A comparison of the magnitudes obtained here with those given by Hagen and Rubens[1] for the same metals shows in some cases excellent agreement, in others a marked difference—as is to be expected.

Fuchsin.—In order that the test for the agreement between calculated and observed reflection coefficients might be as comprehensive as possible it was thought advisable to test a mirror of fuchsin, this being a semi-absorbing medium, whose reflection coefficient is low compared with the more strongly absorbing metals. The mirror was produced by flowing

[1] E. Hagen and H. Rubens, Ann. d. Physik, 1, p. 352, 1900.

an alcoholic solution of fuchsin over a glass plate and allowing it to dry evenly. After a few trials a satisfactorily brilliant and plane surface was obtained upon which the observations were made. The results from this mirror (Table VI., Fig. 4), to obtain which equations (2) instead of (3) were necessarily used, agree very closely with those given by Cartmel[1]

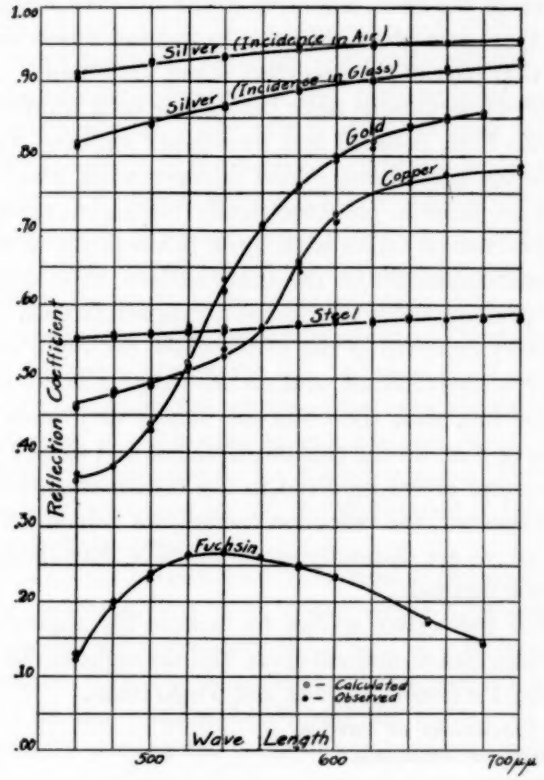

Fig. 4.

TABLE VI.

Fuchsin.

Wave-length (μμ).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.056	.720	.760	.121	.130
480	1.204	.876	1.055	.193	.200
500	1.545	.825	1.275	.230	.238
520	1.860	.739	1.375	.261	.265
540	2.150	.626	1.345	.267	.270
560	2.400	.498	1.195	.261	.260
580	2.555	.378	.966	.247	.250
600	2.660	.252	.683	.232	.235
650	2.438	.055	.135	.176	.170
680	2.231	.025	.056	.146	.142

[1] Cartmel, W. B., Phil. Mag., 6, p. 213, 1903.

whose observations were made on films from the same sample of fuchsin as here used. The curve shows the effect of an absorption region whose maximum lies at about 540$\mu\mu$.

There is general agreement between the calculated and observed values of the reflection coefficients though not quite so satisfactory as with the metals. Owing to the small intensity of the reflected light, particularly in the violet, there was great difficulty in making observations and the agreement in that part of the spectrum is not very close.

Conclusions.—Observations have been made on a number of different substances, with both polished and unpolished surfaces, the magnitude of whose reflection coefficients varied between wide limits (.12 to .96). The reflection coefficients, photometrically measured, agree in all cases within the experimental errors with those theoretically determined from polarimetric measurements on the same surface, thus showing the sufficiency of the present theory to account for these relations. The greatest deviations usually occurred in the violet where the small intensity made accurate observations difficult, and on the steepest slopes of the curves.

Owing to the fact that the observed values vary more widely from the smooth curve than do the calculated values and since a consideration of the experimental errors involved in the measurements indicates that this is to be expected, the reflection coefficients calculated from polarimetric observations are distinctly more accurate than those observed by any photometric method.

In conclusion the writer wishes to express his appreciation for the many helpful suggestions and aid given him throughout the course of this investigation by Professors Skinner and Tuckerman.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA,
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RADIANT EFFICIENCY OF INCANDESCENT FILAMENTS.

By W. E. Forsythe.

OF all the ways in which "efficiency" might be defined, there are two ways which suggest themselves first, the one being purely physical and the other physiological. The first is what has been called "Radiant Efficiency" and is defined as the ratio L/R, where L is the visible radiant energy and R the total radiant energy. The second is the one used by the engineer, that of candle power per watt, which means how much "light" in some arbitrary unit (candle-power) is obtained for each watt of power consumed. This efficiency, to be sure, will depend to a certain extent on the method used to measure the candle power. There has been a great deal of recent discussion concerning the "efficiency" of some of the new metallic filament incandescent lamps, some attributing the increased efficiency to increase in the operating temperature, while others say that it is due to selective radiation in the visible region. The object of the present paper is a determination of the efficiencies defined above as a function of the temperature.

Radiant efficiency as here defined, is a definite physical quantity, but in the various determinations of it, many errors have been introduced due to the methods used to measure either L or R. In the first attempts to measure radiant efficiency of incandescent lamps, the total energy was assumed equal to the wattage input and the energy in the infra-red region measured by putting the lamp in a calorimeter with transparent sides and measuring the rise in temperature of the water in the calorimeter. As this assumes that the energy in the visible region passes out of the calorimeter, a correction must be made for the amount of light absorbed by the water. Merritt[1] with this method obtained from 3.6 per cent. to 7 per cent. as the radiant efficiency of the carbon lamp. However, it is known that the water cell does not absorb all the infra-red radiation, so that his values would on this account be too high. Later Russner[2] attempted to measure the radiant efficiency of incandescent lamps by this method, using in place of a water cell a solution of a double sulphate of iron and ammonia which he says absorbs all the infra-red radiation and transmits all the visible radiation. He obtained as the

[1] American Journal of Science, Vol. 37, p. 167, 1889.

[2] Phys. Zeitschr., *8*, p. 120, 1907.

radiant efficiency 0.6 per cent. for carbon and 2.3 per cent. for tantalum, which are much lower than other values. There seems to be some question[1] as to whether his lamps were at normal brilliancy. However, in no case does this method give the radiant efficiency above as defined, as the total energy measured would not be equal to R, the energy radiated from the filament, but would contain also the energy lost by conduction, etc.

Radiant efficiency can be measured by determining an energy curve of the filament studied and taking the ratio of the area under the curve in the visible region to the total area under the curve. This method would require a great number of readings to obtain the curve and also a troublesome correction, due to the absorption bands in the infra-red. Again, radiant efficiency may be determined by measuring the total energy radiated by one of several methods and then measuring the energy in the visible region from corrected observations of the amount of energy transmitted by a water cell. Houston[2] made use of this method, measuring the visible portion of the energy transmitted by the water cell by a method which will be described later.

Another method is that due to Ångstrom[3] which involves dispersing the total radiation, screening off the infra-red portion, then recombining the visible portion and photometrically balancing it directly against the direct radiation from a similar source. If now the photometer screen be replaced by a thermopile, the energy in the two sources can be compared. A modification of this method has been used[4] to measure the radiant efficiency of the carbon filament as a function of the wattage input. As the temperature was not measured, a comparison cannot be made with the results obtained in this paper, except for normal brilliancy, for which point the value of the radiant efficiency as obtained was 2.6 per cent.

Houston (loc. cit.) measured the radiant efficiency of the tungsten, tantalum and carbon, by measuring the total radiation (apparently through the walls of the incandescent bulb) with and without a water filter, thus determining the percentage of the total radiation transmitted by the filter. Then with a spectrometer having a glass prism and a Rubens linear thermopile in place of cross-hairs, he determined what fraction of the energy, which was transmitted by the water cell, was in the visible spectrum. He assumed that the visible spectrum ends at 0.76μ. By measuring the fraction of the light transmitted by the water cell, he was able to compute L/R, the radiant efficiency. He

[1] W. Voege, Phys. Zeitschr., *8*, p. 306, May, 1907.

[2] P. R. S. of Edinburgh, Vol. XXX., Part 7, 41.

[3] Astrophys. Jour., XV., 223, 1902.

[4] Mendenhall, Phys. Rev., XVII., 1903.

obtained as the radiant efficiency of carbon 2.9 per cent., of tantalum 6.5 per cent. and of tungsten 7.5 per cent. This method seems to be liable to two errors, both of which would give too high values for the efficiency. In the first place, the glass walls of the lamp bulb would surely cut down the total radiation, thus making R too small, and the efficiency too high. Again, according to Coblentz,[1] the glass prism and lenses would not transmit visible and infra-red radiation in the same proportion, but would transmit less in the infra-red region which would result in making the observed efficiency too high. Thus we should expect his results to be too high, the error increasing with the temperature of the source.

Nyswander[2] attempted to determine the radiant efficiency of tungsten from measurements of the areas of a normal energy curve and obtained from 1.5 per cent. to 2 per cent. for this efficiency. He stated that his results were too low due to several errors, but made no attempt to correct for these errors.

As was pointed out above, radiant efficiency is a purely physical efficiency and has but little relation to the other efficiency of "candle power per watt." The question is often asked whether it is worth while to measure radiant efficiency as here defined. This is a purely physical quantity which shows the relative distribution of energy in what is spoken of as the visible and the infra-red spectrum, and if for no other reason, would be of interest. The location of the dividing line between the visible and the infra-red radiation is somewhat artificial but it has been generally agreed to take this at wave-length 0.76μ. It might well be that this could be set at a smaller wave-length and still not very much affect the amount of "light" sent out, still the visible spectrum certainly extends to the A line, if not beyond, and when we speak of the energy in the visible spectrum it should extend as far as the average eye is capable of being affected. Then too, for comparison with other work, the limits should be chosen at a definite position. To say that this should be taken at a smaller wave-length, as was done by Gage,[3] because such a change does not alter the amount of "light" radiated by an appreciable amount as compared to the amount that the energy is altered, seems to rob this efficiency of its meaning. As there is no reason why merely changing the position of the screen with a straight edge should give values that would correspond to the physiological definition of efficiency, we might just as well, if that be

[1] Infra Red Spectrum, Carnegie Publication, Vol. 3–4.

[2] PHYS. REV., Vol. 28, p. 438, 1907.

[3] PHYS. REV., Vol. 33, 2, p. 111, 1911.

the object in view, adopt some such method as that suggested by Fery,[1] which was to use a screen, not with a straight edge, but having a curved edge and the curve being so chosen as to cut down the radiation at each wave-length in inverse proportion to the effect of the radiation at this wave-length on the average eye. This screen would have zero height between 0.5μ and 0.6μ depending upon the intensity of the source and would cut off all the infra-red and ultra-violet radiation, its shape being determined by the visibility curves as found experimentally. Fery did not use this screen but used instead a solution of copper acetate which he found to have a transmission curve similar in form to the visibility curve of the eye. The relative candle powers of different sources as determined by Fery with this method, agree well with values as determined by photometry. A lower limit should also be chosen, but the amount of energy in the ultra-violet region is so small that it can be safely neglected.

To obtain the radiant efficiencies given in this paper, a modification of Ångstrom's method was used. To avoid the difficulty of maintaining two sources exactly alike the set up was so arranged, by using a bent-arm spectroscope, that the light after passing through the spectroscope could be balanced photometrically against the original source. The photometer screen was then replaced by a Rubens thermopile, connected with a Broca galvanometer, so adjusted as to have a sensibility of 150 scale divisions on a scale one meter away for a candle at one meter. As the total radiation contained on the average about twenty times as much energy as that in the visible, the amount of energy falling on the thermopile on the "total" side was cut down by placing between the source and the thermopile a rotating sector, the size of the openings in the sector being chosen such that the deflection of the galvanometer was about the same as that obtained with the visible radiation.

There are several sources of errors, some of which can be corrected for, while others must be guarded against. The slit in the spectroscope must not be too wide or there will be an overlapping of the infra-red radiation in the visible, thus making the radiant efficiency too high. It was assumed that with the width of the slit which permitted one to observe the principal Fraunhofer lines in the solar spectrum, this error would be negligible. The slit width as thus determined was about 0.5 mm. The greatest source of error would be due to the screen used to cut off the infra-red radiation not being exactly in position. This screen was set before each run by reflecting sunlight through the spectroscope. It is thought that most of the variations in the final results are due to

[1] Jour. d. Phys., Vol. 7, p. 632, 1908.

this error and to an error in bringing the filament to the same temperature each time. Precautions were also taken to prevent radiation from the walls of the lamp or other lights in the room from falling on the thermopile. In addition a correction[1] of 1.7 per cent. of the efficiency must be added due to the selective reflection of the silver mirror used in the bent-arm spectroscope, and also there is a correction of 7.8 per cent.[2] to be added due to the fact that the fluorite window used on the lamp to transmit the total radiation was not equally transparent for visible and total radiation. Thus we have a total correction of 9.5 per cent. of the efficiency to be added.

The filaments whose radiant efficiencies were to be measured, were mounted in a special water-cooled brass case with two windows at right angles to each other for making the observations. To obtain the vacuum a Boltwood mercury pump was used, which was allowed to run for several hours. To remove the last traces of the gases another filament, mounted in the base of the can, was burned for a half an hour before the other was started. With this arrangement, if the current through the filament was kept constant, both the temperature of the filament as observed with the optical pyrometer and the voltage across the filament would remain constant. This was taken to indicate good working conditions. In making the observations, the light in the visible and the total spectrum was first balanced photometrically, the photometer being then replaced by the thermopile, and the energy measured in both the visible and the total radiation as pointed out above. This was repeated for different temperatures. From these readings, applying the corrections mentioned above, the radiant efficiency could be calculated.

The temperatures of the filaments were measured with a Holbourn-Kurlbaum optical pyrometer sighted through the fluorite window, the pyrometer lamp being set for the desired temperature and the current through the filament varied until the two appeared to be at the same temperature. This, to be sure, gave black body temperatures, which for the tantalum, tungsten and carbon were corrected to actual temperatures by the wedge method described by Professor Mendenhall.[3] As the temperatures of the filaments were measured through the fluorite window, a correction was necessary for the light that was absorbed and reflected by this window. This correction was easily determined by measuring the temperature of some steady source with the pyrometer, and then the apparent temperature of the same source when sighted through the

[1] Ingersoll, PHYS. REV., Vol. 17, 1903.

[2] Mendenhal, loc. cit.

[3] Astrophys. Jour., Vol. 33, 2, March, 1911.

fluorite window. The Nernst glower was used for the steady source, the correction amounting to about 12° C. at 1500° C. and about 25° C. at 2000° C. In Table I. are given the results for the efficiencies together with the black body and true temperatures.

TABLE I.

RADIANT EFFICIENCY.

Temperature.									
Black Body. $\lambda=0.658\mu$	True.							Mean.	Corr. Mean.
				Tungsten.					
1558	1646	2.9	2.86	2.06	2.55		2.72	2.64	2.89
1698	1805	3.82	3.6	3.29	3.85		3.45	3.60	3.94
1852	1968	3.9	4.8	3.87	4.2	4.22	4.2	4.19	4.59
2000	2135	5.07	5.08	5.23	5.4	5.5	5.25	5.26	5.76
2164	2314	6.35	5.8				6.77	6.31	6.91
				Tantalum.					
1558	1662	1.62	1.54	1.38	1.56	1.56		1.53	1.68
1698	1828	2.80	2.27	2.67	3.06	2.92		2.74	3.00
1852	2012				4.85	4.67		4.76	5.22
				Gem.					
1558		1.8	1.8			1.5		1.7	1.86
1698		2.1	2.2	2.0	2.1	2.1		2.1	2.3
1852		2.7	2.6	3.1	3.0	2.7		2.8	3.1
2000					4.3	4.5		4.4	4.8
				Carbon.					
1515	1550	1.3	1.0	1.1	1.0			1.1	1.2
1720	1762	2.0	1.7	1.6	2.1			1.8	2.0
1815	1860	2.2	1.9	3.1	2.7			2.5	2.7
1945	1995	3.3	2.8	3.1	4.0			3.3	3.6
2025	2082			4.0	4.6			4.3	4.7
2122	2185			5.3	5.9			5.6	6.2

The tungsten filaments, obtained from the General Electric Co., were from a series lamp intended to take about 7.5 amperes for normal brilliancy. The diameter of the filaments was about 0.32 mm. The tantalum filaments were strips 1 mm. wide and about 0.3 mm. thick, rolled from sheets of tantalum obtained from Siemens and Halske. The carbon filaments were taken from a 190-watt Peerless lamp and were about 0.3 mm. in diameter. The metalized filaments furnished by the General Electric Co. were about a square millimeter in cross section. All the filaments were about 2.5 cm. in length.

Temperature of the Filaments in the Lamp Bulbs.—There have been many attempts to determine the temperature of the different incandescent filaments at normal brilliancy, there being quite a wide variation in the different results. For the most part, these determinations are based upon empirical laws which attempt to state the relation between the temperature and either the resulting candle power, resistance of the filament, or the voltage drop or current through the filament. In Table II. are given the different results together with the method used. To measure directly the temperature of a small filament either with a thermo-couple or an optical pyrometer is very apt to lead to wrong results. In either case the results would be apt to be too low, due in the first case to the temperature being lowered by the conduction of the thermo-couple and in the second case to the refraction of the lamp bulb and also to the very small surface that one must compare with the pyrometer lamp filament.

The black-body temperature of the different filaments given in this paper were measured in a manner similar to that used by Waidner and Burgess,[1] excepting that the lamp filaments being tested were balanced photometrically against a Nernst glower which permitted the temperature measurements to be carried much higher than with the carbon strip. In measuring the temperatures the image of the pyrometer lamp filament and the image of the lamp filament being tested were both projected on the same spot of the glower and the black-body temperature of the glower taken when the black-body temperature of the glower was the same as that of the lamp. To obtain monochromatic radiation the same kind of red glass was used before both eye pieces. This method requires a correction for the lens used to project the image of the Nernst glower on the lamp filament as well as for the front wall of the lamp bulb. These corrections were determined in the same manner as the corrections for the fluorite window. The black-body temperatures thus obtained were reduced to true temperatures by the method mentioned above. In Table III. are given results for the temperature as obtained from the different lamps when operated at the voltage or watts marked as normal.

When measuring temperatures above 1500° C., the limit so far attained with the gas thermometer, one must either use a thermo-couple or radiation pyrometer. Either of these must be calibrated in terms of some fixed point, the melting point of palladium or platinum being usually chosen. Now as there is a disagreement as to the exact value of these melting points, one should always mention the value taken. Again, as the black-

[1] Bulletin of the Bureau of Standards, Vol. 2.

TABLE II.

Author.	Year.	Filament.	Temperature (Centigrade).	Watts per Candle Power.	Method.	Reference.
Weber	1892	Carbon	1565 to 1580		Radiation law.	
Lummer & Pringsheim	1899	Carbon	1620 to 1827	3.5	Bolometer.[1]	Ver. d. D. Phys. Ges., Vol. 1, 1899.
Janet P.	1898	Carbon	1610 to 1720		Relation between power consumed and temperature.	Comptes Rendus, V., 126, p. 734, 1898.
Waidner & Burgess[2]	1906	Carbon	1860	3.1	Photometric balance against carbon strip.	Bulletin Bureau Standards, V. 2.
		Tantalum	1865	2.0		
		Tungsten	2135	1.0		
Grau[3]	1907	Carbon	1660	3.5	Photometric balance against iridium strip.	Elektrotechnik und Maschenenbau, April, '07.
		Tungsten	1850	1.14		
Morris, Stroud & Ellis[4]	1907	Tungsten	1727		Stefan's law and matching carbon at 3.5 w./c.p.	The Electrician, Vol. 59, Aug., 1907.
		Tantalum	1577			
Le Chatelier & Bourdouard		Carbon	1800		Absorbing pyrometer.	High Temp. Meas., p. 171.
Coblentz, W. W.	1909	Carbon met.	1570		Energy curve and platinum constant.	Bulletin Bureau of Standards, Vol. 5, p. 339.
		Tantalum	1670			
		Tungsten	1810			
Jolley[5]	1909	Carbon	1575 to 1657	3.5	Empirical relations.	The Electrician, Vol. 63, 1909.
		Gem	1632 to 1717	3.1		
		Tantalum	1427 to 1737	1.66		
		Tungsten	1517 to 1807	1.25		
Fery and Cheneveau[6]	1909	Carbon	1780	3.4	Absorbing pyrometer.	Bull. Soc. Int. Electricians, '09.
		Tungsten	1875	1.21		
Pirani[7]	1910	Tantalum	1975	1.43	Thermo-couple between twisted filaments and extrapolating relation between temperature and power.	Ber. d. D. Phys. Ges., No. 7, p. 301, 1910.
		Tungsten	2060	1.25		
This work[8]		Carbon	1820	3.04	Photometric balance against Nernst glower for black body temperature and true temperature from wedge.	
		Carbon	1845	2.72		
		Gem	1865	2.32		
		Tantalum	1947	1.52		
		Tungsten	2032	1.12		

TABLE III.

No.	Lamp.	Candle Power per Watt.	Temperature.		Style of Lamp.
			Black Body. $\lambda=0.658\mu$	True.	
1	Tungsten	0.811	1863	1980	Old style 25 watt.
1	Tungsten	0.821	1865	1982	Wire filament 25 watt.
1	Tungsten	0.850	1887	2008	Old style 40 watt.
1	Tungsten	0.890	1895	2020	Wire filament 40 watt.
3	Tungsten	0.914	1900	2025	Wire filament 60 watt.
3	Tungsten	0.895	1915	2035	Old style 100 watt.
1	Tungsten	0.944	1920	2040	Wire filament 100 watt.
2	Tantalum	0.513	1727	1862	25 watt.
2	Tantalum	0.561	1750	1890	40 watt.
2	Tantalum	0.655	1795	1945	100 watt.
2	Carbon	0.329	1775	1820	60 watt.
3	Carbon	0.369	1802	1847	120 watt.
3	Carbon	0.368	1798	1843	190 watt.
2	Gem	0.430	1865		50 watt.

body temperature of any substance depends upon the wave-length used the wave-length should be specified along with the black-body temperature. In very few of the values given in Table II. was there any mention of either the temperature scale or the wave-length used. For this work the pyrometer was calibrated by the sector method,[9] taking the melting point of palladium as 1549° C. and that of platinum as 1755° C. c_2 of Wien's equation was taken as 14,500, while the wave-length used with the optical pyrometer was 0.658μ.

[1] The wave-length for maximum energy obtained from energy curve and temperature obtained from Wien's law — $\lambda_m T$ = constant. Assumed that the constant for carbon would lie between that for black body and for platinum.

[2] Black-body temperatures — $\lambda = 0.66\mu$. No mention is made as to whether they correct for the absorption of the lens and the walls of the lamp bulb.

[3] Black-body temperatures. Does not give wave-length used. No mention is made of corrections for absorption of lens or walls of lamp bulb.

[4] Assumed that the temperature of the carbon at 3.5 watts per candle power was 1727° C. Obtained the constant for Stefan's law by bringing other lamps to color match with the carbon.

[5] Measured candle power and watts and used empirical laws connecting one or the other of these with the temperature. Obtained constants of equations by references to other work.

[6] Black-body temperature for carbon, wave-length not given. Black-body temperature of tungsten measured and reduced to true temperature by assuming that tungsten behaved the same as platinum. This would give them a black-body temperature of the tungsten of about 1650° C. which is much lower than other values. As was pointed out above, one would expect a low value for this temperature as measured directly with the pyrometer.

[7] These values have been reduced to a scale with the melting point of palladium at 1549° C.

[8] Black-body temperature for the gem.

[9] Mendenhall, PHYS. REV., Vol. 33, 1, 1911.

The candle power per watt as a function of the temperature was measured for a hundred and ninety watt carbon, a hundred watt tungsten an eighty watt tantalum and a fifty watt gem lamp. These temperatures were first measured as described above, and then the candle power measured with a Lummer-Brodhun photometer, in terms of a standard sixteen candle power carbon lamp obtained from the Leeds and Northrup Co. No very great accuracy can be claimed for the candle power at the high temperatures, due to the very great difference in color between the standard and the lamps tested. These lamps did not appreciably deteriorate when used at this high voltage for the time necessary to make the measurements, as they gave the same candle power per watt when again tested at normal brilliancy. In Table IV. are given the results for this work.

TABLE IV.

Volts.	Current.	Temperature.		Candle Power per Watt.
		Black Body. $\lambda = 0.658\mu$.	True.	
		Carbon Lamp.		
70	1.13	1435	1465	0.057
80	1 29	1526	1560	0.118
90	1.44	1610	1646	0.176
100	1.61	1692	1743	0.278
110	1.76	1780	1824	0.360
120	1.92	1862	1910	0.477
130	2.10	1942	1992	0.662
140	2.29	2012	2067	0.837
		Gem Lamp.		
80	0.335	1578		0.138
90	0.368	1600		0.206
100	0.400	1742		0.296
110	0.430	1829		0.392
120	0.460	1925		0.508
130	0.491	1995		0.653
140	0.521	2080		0.806
		Tantalum Lamp.		
70	0.530	1522	1622	0.195
80	0.585	1600	1711	0.289
90	0.635	1672	1798	0.382
100	0.689	1740	1879	0.502
110	0.735	1805	1955	0.637
120	0.785	1868	2031	0.746
130	0.820	1929	2101	0.902
140	0.884	1988	2173	1.020

Tungsten Lamp.

70	0.705	1612	1708	0.303
80	0.765	1712	1818	0.425
90	0.826	1783	1895	0.563
100	0.880	1840	1955	0.720
110	0.930	1906	2025	0.885
120	0.978	1970	2096	1.100
130	1.025	2026	2160	1.210
140	1.070	2075	2215	1.390

In the curves in Fig. 1 are shown graphically the variation of the radiant efficiency as a function of the temperature and also the candle power per watt as a function of the temperature for the different lamps. For comparison the radiant efficiency of a black body computed from Wien's equation, has been plotted along with the other radiant efficiencies. It will be noticed that the curve showing the efficiency of the carbon runs

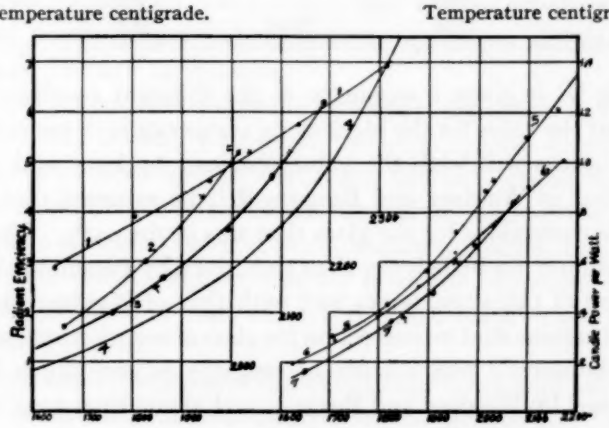

Fig. 1.

Radiant efficiency, (1) tungsten, (2) tantalum, (3) carbon, (4) black body. Candle power per watt, (5) tungsten, (6) tantalum, (7) carbon. (Arrows indicate normal brilliancy.)

almost parallel to that for the black body, being always above it, while the efficiency curve for the tungsten crosses that for the black body at about 2200° C. The tantalum curve at the lower temperatures lies between the tungsten and carbon but crosses the tungsten curve at about 2000° C. The curves showing the variation of the candle power per watt as a function of the temperature lie in a different manner, the tantalum curve in this case lying between the other two for the greater part of the range. A comparison of the two sets of curves would indicate that the distribution of energy in the visible region for the three filaments was entirely different, the tantalum being much richer in the long wave-

lengths as compared with the tungsten. Thus the increased efficiency of the tungsten lamp over that of the tantalum is seen to be due both to its increase in temperature of operation and also to selective radiation in the visible region. Owing to the different arrangements of the filaments in the carbon and the metal filament lamps it is unfair to the carbon lamp to compare horizontal candle power. The value of the radiant efficiency for carbon agrees well with the value found by Professor Mendenhall (loc. cit.), his value being 2.6 per cent.

TABLE V.

Lamp.	Normal Temperature.		Radiant Efficiency.	Candle Power per Watt.
	Black Body. $\lambda = 0.658\mu$.	True.		
Tungsten.....	1912	2032	5.2%	0.890
Tantalum.....	1795	1945	4.3	0.655
Gem.....	1865		3.2	0.430
Carbon.....	1800	1845	2.5	0.368

In Table V. is given a summary of the different results. It will be noticed that the value for the black-body temperature of carbon at normal brilliancy agrees well with the value obtained by Fery and Cheneveau and also that of Waidner and Burgess, if it be assumed that neither of them made corrections for the glass that was in the path. The temperature of tantalum is a little lower than that given by Pirani, the black-body temperature of tantalum agrees well with that of Waidner and Burgess, if we again assume that no correction for glass absorption was made. The value of the normal temperature of tungsten is very much lower than that obtained by Waidner and Burgess, and about 1 per cent. lower than the value obtained by Pirani.

SUMMARY.

The radiant efficiency, black-body temperature and true temperature at normal brilliancy have been determined for carbon, tantalum and tungsten, and the radiant efficiency and black-body temperature of the gem lamp at normal temperature have also been determined. In addition to this, the radiant efficiency and the candle power per watt as a function of the temperature, have been determined for the above lamps.

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ONE-WAVENESS IN WIRELESS TELEGRAPHY; PSEUDO-IMPACT EXCITATION.[1]

By Shunkichi Kimura.

I.

THE characteristics of impact excitation are the fulfillment of one-waveness, at the original wave-length of the secondary system and the absence of the swing of energy between the secondary and primary systems, so that oscillations continue alone in the secondary system, with its own low decrement. In pseudo-impact excitation, the latter characteristic is the result of the former, since at one-waveness there may not be any beat phenomena which may cause the swing of energy. The large value of the primary decrement is enough to account for rapid quenching of primary oscillations.[2]

Of different processes investigated for impact excitation and excitations taken as impact, we may enumerate the short quench sparks,[3] quench-tubes,[4] mercury vapor sparks,[5] hydrogen sparks,[6, 7] ohmic resistance,[5, 6] and partial sparks,[8, 9] all of which are devices to increase the decrement of the primary system. Interest in this impact excitation and its investigation seems to center very legitimately upon the improvement of coupling, hitherto attained in most cases with a limitation in spark potential in the primary system.

The fundamental condition of impact excitation, namely the fulfillment of one-waveness, is also attained, employed and appreciated[10] in ordinary

[1] Excitation akin to the impact excitation.

[2] If δ_1 and ϑ_1 are the damping factor and logarithmic decrement of the primary system, the time in which the initial amplitude falls to $1/e$, and the number of oscillations during the same time are, respectively,

$$\tau = \frac{1}{\delta_1}, \quad m = \frac{1}{\vartheta_1}.$$

In the history of wireless telegraphy, when we required a great number of m, m was called "Resonanzgrade," and when we require a short time for τ, τ is called "Abklingszeit."

[3] Wien, Jahrb., I, 469, 1908.

[4] Wien, Jahrb., IV., 135, 1910.

[5] Glatzel, Jahrb., II., 65, 1909.

[6] Espinosa de los Monteros, Jahrb., I., 480, 1908. Rau, Jahrb., IV., 52, 1910.

[7] Glatzel, Ann. Phys., 34, 711, 1911.

[8] Rohmann, Phys. Zt., 12, 649, 1911.

[9] Galletti, The Electr., Jan. 20, 1911.

[10] One-waveness is found to give the greatest range, though at a great reduction in coupling and efficiency.

excitation, with many kinds of dischargers,[1] though at lower primary decrement, and a looser coupling. The essentials of impact excitation are here almost realized.

Since the same name is sometimes given to different quantities, I write here the formula in full,[2]

$$K'^2 = K^2 - \left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2,$$

where K' is the *degree of coupling*, ∂_1, ∂_2 are the original logarithmic decrements per complete period of primary and secondary systems, and K is the *coefficient of coupling*, defined by

$$K^2 = \frac{L_{12}^2}{L_1 L_2}.$$

The condition of one-waveness may be written in many ways, namely[3]

$$K'^2 = 0; \quad \text{or} \quad K^2 = \frac{1}{4\pi^2}(\partial_1 - \partial_2)^2; \quad \text{or} \quad L_{12} = \frac{1}{2\pi}\sqrt{L_1 L_2}\cdot(\partial_1 - \partial_2);$$

or

$$\partial_1 = \partial_2 + 2\pi K.$$

Here it is assumed from practical cases that the decrement in the primary with a spark gap is larger than the decrement in the secondary which consists of ohmic and radiation decrements. Of the above condition, the first comes from the theory of coupled systems; the second may be reached from practical trials.[4] Of the third form, we may say that the measurement of wave-length with a wavemeter employs this or less amount of mutual induction at perfect synchronism, and one wave is produced in the instrument, the wave-length of which is read and made the wave-length of exciting circuit. The fourth form is known from the experimental fact that the primary decrement must be much larger than the secondary decrement to effect a good impact excitation; the difference

[1] Fessenden's synchronous gap in U. S. N. Experiment; Austin, Bull. Bur. Stand., 7, No. 3, 1911. Marconi's rotating gap, The Electrician, July 14, 1911. Gaps in series, gaps with air blast have the same function.

[2] The expression of the degree of coupling and its connection with the two coupling waves seem to appear originally in Drude's paper, Ann. Phys., 13, 1904, p. 544.

[3] Pseudo-impact excitation occurs at the value of K'^2 very little less than zero. The value of K here expressed will be called the *critical coupling* and will be understood as the limiting value above which the pseudo-impact excitation is impossible.

[4] For instance, Wien, Jahrb., I., 471. The theory neglects the presence of a spark in the primary system, and hence we are to expect some correction in practice, and this correction is very little. For example, in Wien's measurement (Wien, Ann. Phys., 29, 1909, p. 711, Tables 29, 30) values of K'^2 for $K = 0.013$, were -0.0003 and $+0.000098$, respectively, and in these cases maximum effects in the third circuit or resonator were observed.

between the two decrements being here $2\pi K$, depending on the amount of coupling. The theory of this one-waveness is very simple; we revert to the case of two systems at an extremely loose coupling,[1] and take the condition of isochronism[2] in theory, expecting a certain correction in case of resonance in practice.[3]

The resulting oscillation in the secondary system is the difference between two oscillations with the same frequency but different decrements and gives rise to an amplitude curve tapering at the beginning. The maximum current amplitude attainable in the secondary system depends on the fundamental quantities of the primary and secondary systems and is represented in practical units by

$$I_{\max}(\text{amp.}) = \frac{1}{2} \cdot \frac{\partial_1 - \partial_2}{\partial_2} \left(\frac{\partial_2}{\partial_1}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} \sqrt{\frac{C_1(\text{mf.})}{L_2(\text{mh.})}} \cdot V_f(\text{volt}),^{4}$$

where C_1, V_f are the capacity in microfarads and spark potential in volts in the primary system, and L_2 is the self-inductance in microhenries in the secondary system. Taking this formula for the pseudo-impact excitation and provisionally also for the impact excitation, let us try some calculations with some practical data.[5]

Example I.—One-waveness by large spark ordinary excitation. Given

$$\partial_2 = 0.004; \quad C_1 = 0.01 \text{ mf.}; \quad V_f = 6 \cdot 10^4 \text{ volts}; \quad L_2 = 100 \text{ mh.}$$

Pseudo-impact excitation is attained at $K = 4.5$ per cent.[6] for instance. Then we obtain

[1] Bjerknes, Wied. Ann., 44, 1891, 74; 55, 1895, 121; Zenneck, E. M. S., 584, 1905, L. D. T., 79, 1909. It is interesting to note that the theoretical and practical investigations were directed for some time to the reduction of primary decrement and to the production of maximum potential and current in the secondary system with two coupling-waves at close coupling, and now we have changed the course.

[2] In impact excitation, this condition must also be observed in order to obtain the most powerful oscillations in the secondary system.

[3] In practical trials with ordinary and impact excitations at the condition of one-waveness with maximum effect in the secondary or aerial system, isochronism and resonance (Zenneck, E. M. S., 556) often, though not always, become identical, within the reading-error of a wave meter. Perfect adjustment to one-waveness at maximum effect is not a very easy process, nor of a very definite delineation. Frequency and wave-length are to take account of the correction due to the larger values of decrements; the wave-meter indicates the corrected quantities.

[4] In case of $\partial_1 = \partial_2$ at isochronism, it requires a special treatment in theory, leading to quite different formulæ; as to the production of one-waveness, it is also a door case. This case is excluded in this paper.

[5] The theory of impact excitation will perhaps require the primary decrement ∂_1 to be treated as a function of time, practically to reach an infinitely large value at the end of first half beat time, and not as a constant as in case of ordinary excitation.

[6] Glatzel, Ann. Phys., 34, 711, 1911; Fleming and Dyke, The Electr., March 3, 1911; Wien, Jahrb., 4, 135, 1911.

$$\vartheta_1 = 0.2867;^{1} \quad I_{max} = 273 \text{ amp.}$$

Example II.—One-waveness by short spark impact excitation. Given

$\vartheta_2 = 0.004$; $C_1 = 0.01$ mf.; $V_f = 3 \cdot 10^4$ volts; $L_2 = 100$ mh.

Impact excitation is attained at $K = 13$ per cent.[2] for instance. Then we obtain

$$\vartheta_1 = 0.821; \quad I_{max} = 146 \text{ amp.}$$

Since the above formula, $\vartheta_1 - \vartheta_2$ is nearly equal to ϑ_1, the factor consisting of decrements is almost unity, more so as ϑ_1 is larger than ϑ_2, and with no respect to coupling; therefore, under given constants of primary and secondary systems, the maximum current amplitude is almost proportional to the spark potential in the primary system. If ϑ_2 is made larger, the same factor may differ more from unity. Now, with the condition of one-waveness, we may write,

$$D = \frac{\vartheta_1 - \vartheta_2}{\vartheta_2} \cdot \left(\frac{\vartheta_2}{\vartheta_1}\right)^{\frac{\vartheta_1}{\vartheta_1 - \vartheta_2}} = \frac{x^x}{(1+x)^{1+x}},^{3}$$

where

$$x = \frac{\vartheta_2}{2\pi K} = \frac{\vartheta_2}{\vartheta_1 - \vartheta_2}.$$

This factor is larger as x is smaller, or as ϑ_2 is smaller at constant ϑ_1, or as ϑ_1 is larger at constant ϑ_2. The value of ϑ_2 can not be smaller than the radiation decrement; hence ϑ_1 must be made larger and then the coupling is improved.[4]

Considering the other factor of I_{max}, namely $\sqrt{C_1} \cdot V_f$, pseudo-impact excitation with larger coupling and smaller energy per spark and the same

[1] Measured with Marconi's decremeter, series gaps with 2 mm. spark each, gave $\vartheta_1 = 0.41$ almost constant, single-blasted gap gave $\vartheta_1 = 0.30$, larger for shorter spark, smaller for longer. Coupling must be adjusted every time for one-waveness when sparks or dischargers are changed.

[2] Glatzel, Ann. Phys., 34, 711, 1911; Fleming and Dyke, The Electr., March 3, 1911.

[3] A table of this quantity for values of x which occur in practice is given below.

[4] In similar manner, the time at which the secondary current reaches the maximum value I_{max} may be expressed in terms of x, namely,

$$t = \frac{2}{n} \cdot \frac{\log \vartheta_1 - \log \vartheta_2}{\vartheta_1 - \vartheta_2} = \frac{2}{n\vartheta_2} x \left[\log (1+x) - \log x\right]$$

$$= \frac{2}{n\vartheta_2} \log \frac{1}{(1+x)D}.$$

Hence this time is a function not only of ϑ_1, ϑ_2, or K and ϑ_2, but it changes with the working wave-length. For example:

1. $\vartheta_1 = 0.4$; $\vartheta_2 = 0.08$; $n = 10^6$; $\lambda = 600m$; $t = 10 \cdot 10^{-6}$;
2. $\vartheta_1 = 0.8$; $\vartheta_2 = 0.08$; $n = 10^6$; $\lambda = 600m$; $t = 6 \cdot 10^{-6}$.

In the first case, the maximum amplitude occurs after 5 complete oscillations; in the second after 3.

excitation with smaller coupling and larger energy per spark are something like a balance between two above factors, and it can not be decided at once which factor is more important, best result being expected from the satisfaction of two factors, namely the closer coupling and larger energy per spark in the primary system.[1]

Example III.—Fleming and Dykes' Experiment.[2]

	Given Secondary Decrement.	One-waveness Observed. Per Cent.	Calculated Primary Decrement.	Number of Primary Oscillations till Amplitude Falls to $1/e$.	$x=\partial_2/2\pi K$.	D	$I_{max}/\sqrt{\partial_2}$ Ratios.
Ordinary ..	$\partial_2=0.02$	$K=4.5$	$\partial_1=0.3027$	3.3	0.07074	0.7706	1
Excitation.	$\partial_2=0.18$	$K=6.7$	$\partial_1=0.6049$	1.6	0.4235	0.4204	0.2
Impact....	$\partial_2=0.02$	$K=11$	$\partial_1=0.7111$	1.4	0.0289	0.8787	1
Excitation.	$\partial_2=0.18$	$K=13$	$\partial_1=0.9968$	1.0	0.2203	0.5620	0.2

In the next to last column, the factor of I_{max} depending only on the decrements is calculated. In the last column, these numbers are taken proportional to I_{max}. Anticipating the indication of a heat-working current measurement given below, currents in the secondary system are calculated in ratios, each for two excitations, and these ratios are about the same as those given in Fig. 4 in the authors' paper.

II.

The indication of a heat-working instrument in the secondary system takes a different form from that in case of damped trains of oscillations with one decrement and maximum initial amplitude. In the calculation I take the assumptions which are plausible in practical cases, namely,

$$(\partial_1 + \partial_2)^2 \ll 4\pi^2, \quad \partial_2 \ll \partial_1.$$

The initial value of current amplitude which is equal for the two synchronous oscillations in the secondary system, but which is not a measurable quantity, is

$$I_0 = \pi \frac{L_{12}}{L_2(\partial_1 - \partial_2)} I_{10} = \frac{1}{2}\sqrt{\frac{L_1}{L_2}} \cdot I_{10} = \frac{1}{2}\sqrt{\frac{C_1}{L_2}} \cdot V_f;$$

$$= \frac{\partial_2}{\partial_1 - \partial_2}\left(\frac{\partial_1}{\partial_2}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} \cdot I_{max};$$

[1] So far as theory goes, the advantage resulting from the coupling reaches a limit, when $x = 0$; $D = 1$, max. and limiting value. In practice, values 0.7 to 0.8 are already attained in case of impact excitation.

[2] Fleming and Dyke, The Electrician, March 3, 1911. As I take the data from the abstract, calculation is only half way and regret not having access to their original paper.

where I_{10}, V_f are the initial and maximum current amplitude and the spark potential in the primary system and I_{max} is the maximum current amplitude attainable in the secondary system. The current in the secondary sytem takes the form

$$I = I_0(e^{-\delta_1 t} - e^{-\delta_2 t}) \sin \pi n t,$$

where δ_1, δ_2 are the original factors of damping in the two sytems, and n is twice the isochronous frequency. Therefore the heat produced in the instrument with a low resistance R in one second of time and at ζ sparks per second is represented by

$$Q = \zeta R \int_0^\infty I^2 dt$$

$$= \frac{\zeta R I_0^2}{2n}\left[\frac{1}{\partial_1\left[1+\left(\frac{\partial_1}{2\pi}\right)^2\right]} + \frac{1}{\partial_2\left[1+\left(\frac{\partial_2}{2\pi}\right)^2\right]} - \frac{2}{(\partial_1+\partial_2)\left[1+\left(\frac{\partial_1+\partial_2}{2\pi}\right)^2\right]}\right].$$

Whence we obtain by the above assumptions

$$Q = \frac{\zeta R}{8n}\cdot\frac{1}{\partial_2}\cdot\frac{L_1}{L_2}\cdot I_{10}^2$$

$$= \frac{\zeta R}{8n}\cdot\frac{1}{\partial_2}\cdot\frac{C_1}{L_2}V_f^2 = \frac{\pi^2}{4}\zeta R n\frac{C_2}{\partial_2}\cdot W$$

$$= \frac{\zeta R}{2n}\cdot\frac{1}{\partial_2}\cdot I_{max}^2.$$

Reducing this to the indication of a hot wire ammeter, we have

$$I = \frac{\pi}{2}\sqrt{\zeta n}\sqrt{\frac{C_2}{\partial_2}w}$$

$$= \sqrt{\frac{\zeta}{2n\partial_2}}\cdot I_{max}.$$

This is the usual expression for one group of damped trains, except that I_{max} takes the place of the initial amplitude.

III.

In order to study the results of coupling when K'^2 is positive, negative or zero, I start from the biquadratic Z-equation of Drude,[1] which is

$$Z^4 + Z^2[\tau_1^2 + \tau_2^2 - \tfrac{1}{2}(\vartheta_1 - \vartheta_2)^2] - Z(\tau_1^2 - \tau_2^2)(\vartheta_1 - \vartheta_2)$$

[1] Drude, Ann. Phys., 13, 1904, p. 534.

$$+\left[\left(\frac{\vartheta_1-\vartheta_2}{2}\right)^2+\tau_1^2\right]\left[\left(\frac{\vartheta_1-\vartheta_2}{2}\right)^2+\tau_2^2\right]=K^2(\tau_1^2+\vartheta_1^2)(\tau_2^2+\vartheta_2^2),$$

with

$$y=-\vartheta+i\tau=-\frac{\vartheta_1+\vartheta_2}{2}+Z.$$

Reducing to the notation previously used and at isochronism of the two systems and with the valid assumption, ∂_1^2 and $\partial_2^2 \ll 4\pi^2$,

$$\tau_1=\tau_2=\frac{1}{\pi n};\quad \vartheta_1=\frac{\partial_1}{2\pi^2 n};\quad \vartheta_2=\frac{\partial_2}{2\pi^2 n};$$

$$\tau_1^2+\vartheta_1^2=\tau_2^2+\vartheta_2^2=\frac{1}{\pi^2 n^2}.$$

The above equations then take the form

$$\left\{Z^2+\frac{1}{\pi^2 n^2}\left[1-\frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2\right]\right\}^2=\frac{1}{\pi^4 n^4}\left\{K^2-\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2\right\}=\frac{1}{\pi^4 n^4}K'^2,$$

with

$$y=-\frac{\partial_1+\partial_2}{4\pi^2 n}+z=-\frac{\Delta}{2\pi^2 N}+i\frac{1}{\pi N},$$

where Δ's and N's are the decrements and double frequencies after the coupling, to be found from the solution of above equation.

1. K'^2 positive, with any magnitude. In this case we have

$$Z=\pm i\frac{1}{\pi n}\sqrt{1-\frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2\pm K'},$$

and the required quantities are

$$N=\frac{n}{\sqrt{1-\frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2\pm K'}}=\frac{n}{\sqrt{1\pm K'}},$$

$$\Delta=\frac{N}{n}\cdot\frac{\partial_1+\partial_2}{2}\cdot=\frac{1}{\sqrt{1\pm K'}}\cdot\tfrac{1}{2}(\partial_1+\partial_2),$$

the two coupling waves, each with different decrement.

2. K'^2 zero. Here we have

$$Z=\pm i\frac{1}{\pi n}\sqrt{1-\frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2}$$

and

$$N=\frac{n}{\sqrt{1-\frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2}}=n,\quad \Delta=\frac{N}{n}\cdot\frac{\partial_1+\partial_2}{2}=\tfrac{1}{2}(\partial_1+\partial_2).$$

Here only one wave is produced with nearly the original isochronous frequency, and the decrement is also unique but its value is nearly equal to the arithmetical mean of those of the two systems before coupling.

3. K'^2 negative, small in absolute value. In this case we use the expression of K^2 and we have

$$Z^2 = -\frac{1}{\pi^2 n^2}\left[1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 \pm i\sqrt{\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 - K^2}\right];$$

whence, for small absolute value of K'^2,

$$Z = \pm\frac{1}{2\pi n}\sqrt{\frac{\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 - K^2}{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}} \pm \frac{i}{\pi n}\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}.$$

Therefore the required quantities are

$$N = \frac{n}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}} = n,$$

$$\Delta = \frac{N}{n}\left\{\frac{\partial_1 + \partial_2}{2} \pm \pi\sqrt{\frac{\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 - K^2}{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}}\right\}$$

$$= \tfrac{1}{2}(\partial_1 + \partial_2) \pm \tfrac{1}{2}(\partial_1 - \partial_2)\sqrt{1 - \frac{4\pi^2 K^2}{(\partial_1 - \partial_2)^2}}.$$

Thus there are produced two coincident waves with the same frequency which is nearly the same with those original isochronous frequencies, but these two waves have different decrements. When we treat for small quantities, especially for K^2, these decrements become

$$\Delta_1 = \partial_1 - \frac{\pi^2 K^2}{\partial_1 - \partial_2}; \quad \Delta_2 = \partial_2 + \frac{\pi^2 K^2}{\partial_1 - \partial_2}.$$

Thus the decrement in the primary sytem is very little less than its original value ∂_1, and that in the secondary system very little larger than its original value ∂_2. Such is the pseudo-impact excitation, improved in the resulting decrements by making the primary decrement larger and the secondary decrement smaller.

4. The nature of *impact excitation* may be conceived in the following way as the result of the change in primary decrement. Starting with an inherently large value of primary decrement at a positive value of K'^2, hence with two coupling waves producing beats, the same decrement in-

creases at a rapid rate until it reaches a final large value at the end of one half beat time. Consequently K'^2 passes from its positive phase, through zero to a negative phase, and at the time when the primary decrements reach a final value, the two systems are at the condition of an extremely loose coupling with large ϑ_1 and negative K'^2, with the resulting decrements and double frequency

$$\Delta_1 = \frac{N}{n}\vartheta_1; \quad \Delta_2 = \frac{N}{n}\vartheta_2; \quad N = \frac{n}{\sqrt{1 - \frac{1}{4}\left(\frac{\vartheta_1}{2\pi}\right)^2}};$$

where ϑ_1 is the final value.

Every such transition takes place within a very short time, and the law of change of primary decrement as a function of time is not yet known. As a consequence, the degree of coupling represented by

$$K'^2 = K^2 - \left(\frac{\vartheta_1 - \vartheta_2}{2\pi}\right)^2$$

and the primary double-frequency represented by

$$n_1 = \frac{1}{\pi}\frac{1}{\sqrt{C_1 L_1}}\sqrt{1 - \left(\frac{\vartheta_1}{2\pi}\right)^2}$$

are not constant, but simply decrease to the final values with a law not yet known, and also the time of half beat can not be written in the usual formula in terms of K', assumed constant. The value of ϑ_1 calculated in part I. for impact excitation is only the *equivalent value*, reduced to the case of constant decrement and at zero phase of K'^2. Meanwhile, if we limit ourselves to the final phase at which the primary oscillation ceases, the condition of impact excitation is the same with that of pseudo-impact excitation and therefore we may take the expression I_{max} for the secondary maximum amplitude in part I. to be the same for the impact as well as for the pseudo-impact excitations, with ϑ_1 as the equivalent value in the former and as a constant in the latter. Also the indication of a heat-working instrument, containing ϑ_2 only in part II., may be taken, as it is, in case of impact excitation, and on the assumption $\vartheta_1 \gg \vartheta_2$ in case of pseudo-impact excitation. Distinction between these two excitations may be made only by observing the oscillation curves in the primary and secondary oscillations, but probably not in any other way hitherto known. These two excitations may also be a continuation with a difference only in degree, the phenomena not being fundamentally different, if we can prove the variation of the primary decrement existing also in the ordinary excitation.

TABLE I.

Values of $D = \frac{\partial_1 - \partial_2}{\partial_2}\left(\frac{\partial_2}{\partial_1}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} = \frac{x^x}{(1+x)^{1+x}}$

for the Values of $x = \frac{\partial_2}{\partial_1 - \partial_2} = \frac{\partial_2}{2\pi K}$.

x	D	x	D
0.005	0.969	0.055	0.805
0.010	0.945	0.060	0.794
0.015	0.925	0.065	0.783
0.020	0.905	0.070	0.770
0.025	0.889	0.075	0.762
0.030	0.873	0.080	0.750
0.035	0.857	0.085	0.741
0.040	0.843	0.090	0.731
0.045	0.831	0.095	0.723
0.050	0.816	0.100	0.715

TABLE II.

Values of Critical Coupling, K in Per Cent.

∂_2	∂_1 0.1	∂_1 0.2	∂_1 0.3	∂_1 0.4	∂_1 0.5	∂_1 0.6	∂_1 0.7	∂_1 0.8	∂_1 0.9	∂_1 1.0
	Mean 0.76 Per Cent.	Mean 2.35 Per Cent.	Mean 3.94 Per Cent.	Mean 5.53 Per Cent.	Mean 7.12 Per Cent.	Mean 8.71 Per Cent.	Mean 10.31 Per Cent.	Mean 11.90 Per Cent.	Mean 13.49 Per Cent.	Mean 15.08 Per Cent.
0.005	1.51	3.10	4.70	6.29	7.88	9.47	11.06	12.65	14.24	15.84
0.010	1.43	3.02	4.62	6.21	7.80	9.37	10.98	12.57	14.17	15.76
0.015	1.35	2.94	4.54	6.13	7.72	9.31	10.90	12.49	14.09	15.68
0.020	1.27	2.86	4.46	6.05	7.64	9.23	10.82	12.41	14.01	15.60
0.025	1.19	2.79	4.38	5.97	7.56	9.15	10.74	12.33	13.93	15.52
0.030	1.11	2.71	4.30	5.89	7.48	9.07	10.66	12.26	13.85	15.44
0.035	1.03	2.63	4.22	5.81	7.40	8.99	10.58	12.18	13.77	15.36
0.040	0.95	2.55	4.14	5.73	7.30	8.91	10.50	12.10	13.69	15.28
0.045	0.88	2.47	4.06	5.65	7.24	8.83	10.42	12.02	13.61	15.20
0.050	0.80	2.39	3.98	5.57	7.16	8.75	10.35	11.93	13.53	15.12
0.055	0.72	2.31	3.90	5.49	7.08	8.67	10.27	11.86	13.45	15.04
0.060	0.64	2.23	3.82	5.41	7.00	8.59	10.19	11.78	13.37	14.96
0.065	0.56	2.15	3.74	5.33	6.92	8.52	10.11	11.70	13.29	14.88
0.070	0.48	2.07	3.66	5.25	6.84	8.44	10.02	11.62	13.21	14.80
0.075	0.40	1.99	3.58	5.17	6.76	8.36	9.95	11.54	13.13	14.72
0.080	0.32	1.91	3.50	5.09	6.68	8.28	9.87	11.46	13.05	14.64
0.085	0.24	1.83	3.42	5.01	6.61	8.20	9.79	11.38	12.97	14.56
0.090	0.16	1.75	3.34	4.93	6.51	8.12	9.71	11.30	12.89	14.48
0.095	0.08	1.67	3.26	4.85	6.45	8.04	9.63	11.22	12.81	14.40
0.100	——	1.59	3.18	4.77	6.37	7.96	9.55	11.14	12.73	14.32

The secondary decrement, when the secondary system is an aerial circuit, consists of ohmic and radiation decrements and takes the form[1]

$$\partial_2 = \frac{\lambda(m)}{6 \cdot 10^2 L_2(\text{mh.})} \cdot \left\{ R(\text{ohm}) + K\frac{h^2}{\lambda^2} \right\},$$

where the quantity within the bracket is the total equivalent resistance of the same circuit. The radiation decrement increases as the wave-length is shorter for the same effective height h, while the ohmic resistance may be almost constant and may be kept low. In practical cases this radiation decrement alone may reach as much as 0.1. Since the value of ∂_2 changes with the working wave-length, coupling for one-waveness must be adjusted every time the working wave-length is changed. Table II. gives K in per cent. for one-waveness for various combinations of ∂_1 and ∂_2.[2] We see that for each value of ∂_1, K is less as ∂_2 is increased; but for the values of ∂_1 greater than 0.3, and $0 < \partial_2 < 0.1$, K is almost determined from the values of ∂_1 alone. For a constant secondary system, everything which increases ∂_1 increases the coupling for one-waveness and hence the efficiency is improved, and such may be the same with the improvement of "Löschwirkung"[3] in impact excitation.

IV.

One-waveness at the Receiver.

One-waveness is also important in the receiver as in the transmitter. In order to consider this question we will take one sample diagram shown in Fig. 1, by which couplings and connections will be made manifest and definite.

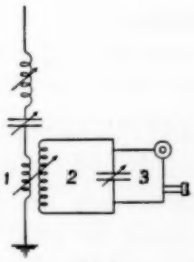

Fig.1.

When the trains of waves arrive at the place of an isochronous receiving aerial circuit and there produce the field represented by $E_{ao}e^{-\delta t}\cos \pi n t$, there are set up oscillations in the aerial circuit, of which energy, one part is transferred to the inner circuit coupled to it, but the other part is partly lost as heat and partly radiated. This radiated energy has the same frequency as the field, and superpose another field on it with a sort of coupling, the result being likely to produce the coupled frequencies both in the aerial circuit and the surrounding field. This can

[1] Rudenberg, Ann. Phys., 25, 1908, 451. The law $k\frac{h^2}{\lambda^2}$ appears to be confirmed experimentally by Austin, in case of k = constant; Bull. Bur. Stand., 7, No. 3, 1911.

[2] The relation (∂_1, K) with parameter ∂_2 are all parallel straight lines, easily extended for larger values of ∂_1 and ∂_2.

[3] Wien, Jahrb., 4, 135, 1911.

be avoided by making the aerial circuit less radiative by inserting an inductance without increasing the resistance in a proportionate amount. By this device the damping of the aerial circuit becomes much smaller than that of the arrived waves, and the resulting current oscillations in this circuit may be represented by

$$I_1 = I_{10}(e^{-\delta t} - e^{-\delta_1 t}) \sin \pi n t,$$

where δ_1 is the damping of the circuit and the maximum amplitude is represented by[1]

$$I_{10} = \frac{E_{a0}}{2\pi n L_1(\delta - \delta_1)}.$$

This is one-waveness and takes the maximum amplitude at the time

$$t = \frac{1}{\delta - \delta_1} \log n \frac{\delta}{\delta_1}$$

and

$$I_{1\max} = I_{10} \frac{\delta}{\delta_1} \left(e^{-\frac{\delta_1}{\delta - \delta_1}} - e^{-\frac{\delta}{\delta - \delta_1}} \right).$$

After this time we may write the equation for the aerial circuit in the form

$$I_1 = I_{1\max} e^{-\delta_1 t} \sin \pi n t.$$

In the next circuit 2, which is a closed condenser circuit made to have a very much smaller damping δ_2 than δ_1, and coupled to a one-waveness to the aerial circuit,[2] we may write the potential equation in the form

$$V_2 = V_{2\max} e^{-\delta_2 t} \sin \pi n t,$$

where

$$V_{2\max} = \frac{1}{4\pi n} \cdot \frac{\delta_1 - \delta_2}{\delta_2} \left(\frac{\delta_2}{\delta_1} \right)^{\frac{\delta_1}{\delta_1 - \delta_2}} \frac{1}{C_1} \sqrt{L_1 L_2} \cdot I_{1\max}.$$

And this oscillating potential works upon the next circuit. The circuit 3 consists of the detector and telephone in which the resistance and self-inductance are very large compared to the previous circuit. In this circuit we must make distinction betweeen wave frequency and group frequency. With respect to the wave frequency, circuit 3 is entirely out of tune or aperiodic and hence there is produced only the forced oscillations. The detector here works by rectification[3] and, if this rectification

[1] Bjerknes, Wied. Ann., 55, 1895, 121.

[2] At close coupling, presence of two coupling waves may be observed, see also L. W. Austin, Bull. Bur. St., Vol. 7, No. 2, 1911.

[3] This is one explanation, and in mineral detectors, some much more complicated action is suspected, though it takes no definite shape. So far as known, all forms of present detectors possess the property of polarity, whether electric rectification or magnetic hysteresis. The sense of rectification is constant while the potential working on the mineral detectors is

is perfect, current which passes through the telephone coil may be represented by

$$I_3 = I_{3\max}\left[\int_0^{1/2n} + \int_{3/2n}^{5/2n} + \int_{7/2n}^{9/2n} + \int_{11/2n}^{13/2n} + \cdots\right] e^{-\delta_2 t}\cos \pi n t dt,$$

where

$$\measuredangle (I_3, V_2) = \frac{\pi}{2},$$

$$I_{3\max} = \frac{V_{2\max}}{\sqrt{R_3^2 + (\pi n L_3)^2}} = \frac{1}{\pi n L_3}\cdot V_{2\max}.$$

This current is taken as a simple summation for this reason. The frequency of the oscillatory current is of the order 10^6 and that of the telephone diaphragm of the order 10^3; hence the most part of the oscillations in one train of currents passes through the telephone coil while the diaphragm is making one quarter vibration, from its normal position to the extreme amplitude. When the above integrations are made, the current through the telephone coil is represented by

$$I_3 = \frac{1}{\pi n} I_{3\max}\left[\frac{1}{2\pi}\partial_2 + \frac{e^{\frac{1}{2}\partial_2}}{e^{\frac{1}{2}\partial_2} - 1}\right]$$

$$= \frac{1}{\pi n} I_{3\max}\left(\frac{2}{\partial_2} + \frac{1}{2} + \frac{1}{2\pi}\partial_2\right)$$

$$= \frac{2}{\pi n \partial_2}\cdot I_{3\max},$$

where ∂_2 is the decrement of the circuit 2. Thus there is apparently an accumulative action of current wave-trains in the receiver, but this is due to the mechanical slowness of the telephone diaphragm and does not extend its integrating action to the group frequency as in a heat-working instrument. It is the above current which is indicated in a moving-coil galvanometer placed in the circuit 3, and it is this current which produces the amplitude of vibration of the diaphragm, and which may be smaller and still keep up audibility, according as the group frequency approaches the natural frequency of the vibrations of the diaphragm. As regards the group frequency ζ, the intermittent current I_3 flows through the telephone coils, ζ times per second and gives rise to the note of ζ. For this frequency we have mechanical resonance and a certain kind of coupling. For the former, the damping of the diaphragm is very great; consequently it vibrates with forced group frequency. If the lowest natural frequency

large, but when it is small we observe reversals, the galvanometer deflecting in opposite senses irregularly, and the tone audible in the telephone while the galvanometer shows no deflection at all.

of the diaphragm is p, and the group frequency ζ, the intensity of tone produced will be proportional to

$$\frac{1}{(p^2 - \zeta^2)^2},$$

which expression of course excludes the idea of damping.[1] As to the kind of coupling, transfer of energy from the diaphragm back to the circuit 3 is in the form of an opposite current in the same double frequency ζ due to the vibration of an iron plate in front of the pair of permanent magnets, and there is need of something like one-waveness, by the adjustment of coupling. Such is already attained by Pickard's adjustable telephone, in which the magnet and coils are adjusted to a proper distance from the diaphragm, and the effect of this adjustment on the intensity of tone is something marvellous, while in some other telephone such adjustment is made at the manufacturing place. Thus, in the telephone for use in wireless telegraphy, mechanical resonance of the diaphragm to the group or spark frequency and the proper coupling or distance between it and the magnet, are important items of manufacture, while its resistance is immaterial within a wide limit, since it is negligible in comparison with its inductance.

[1] M. Wien, Ann. Phys., 18, 1049, 1905.

AN INTERPRETATION OF CURL AND DIVERGENCE IN TERMS OF VECTOR-GROWTH.

By Frederick Slate.

THE essential steps preliminary to the classification and description of stationary vector fields are now usually presented substantially as follows:

1. The idea of gradient for a given scalar field is introduced and brought into relation with potential.

2. The test is formulated whether or not a scalar field can be specified, of which a given vector field would be the gradient. This yields the criterion for the occurrence of a single-valued scalar potential.

3. The curl of a vector field is then definitely set up as a measure of departure from that criterion.

4. This measure of curl is made to appear in two forms, whose relation leads to the possible equivalence of integrations round a bounding curve and over the enclosed area (Theorem of Stokes). The theorems of Gauss and Green are then supplementary in their idea. They and the measure of divergence can be put first, however, if made a part of the discussion of potential, as they often are.

The reasons for adopting such a sequence lie obvious in the history of the subject. Curl is made, so to speak, an offshoot of potential because the latter type of field was earlier prominent; and because the equality of $\partial \mathbf{V}_y/\partial x$ and $\partial \mathbf{V}_x/\partial y$[1] gave a natural starting-point for the general expression of their difference. But the actual free use of curl and divergence in electro-magnetism has been leading toward revision that may exhibit the kernel of these matters differently; and this general aim guides the present paper, whose special intention is to emphasize another possible logic in the order of ideas, and in their foundations. The discussion is limited to steady fields.

If we approached without bias the question of classifying vector fields we might begin with the simple case—the uniform field, in which the resultant vector is constant in direction and magnitude. For such fields, too, as we know, curl and divergence are everywhere zero. It is known, moreover, that two pure types of modified field can be specified: one

[1] With some apology for the hybrid notation. But it seemed desirable to accentuate the central vector quantity, even where the equations are algebraic.

exhibiting curl alone, and the other divergence alone; while, further, any steady vector field can be analyzed, for purposes of quantitative description, as a combination of the two pure types. If our thought follows this sequence, less artificial than some others, it is plain that we coördinate curl and divergence on equal footing in their origin; each represents a particular condition of departure from the same original uniformity. At the same time, when taken in superposition, they describe the local arrangement of vector throughout any field adequately, and with simplifications due to the mode of segregation. The question is approached as here indicated.

In any uniform field the projection of the vector in any given direction is equal at all points. If then any base-point be chosen within such a field, and any definite radius-vector be drawn, all along that line there will be equal radial projections of the field-vector; and, of course, equal projections perpendicular to that radius-vector. But we must note that the conventions attached to the radius-vector will result in opposite sign for both radial and perpendicular projections, if we prolong the radius-vector backward through the origin; that is, in effect, use a pair of radius-vectors at an angle of 180°. Any non-uniform field will in general show change in the radial component or in its perpendicular, or in both, as we pass from point to point along any given radius-vector; and this brings the immediate suggestion that we should decipher in the character and the rapidity of such changes the special phases of non-uniformity occurring in any particular case. By following this clew we are in fact led to find in curl and divergence an appropriate and simple measure of the important changes in vector-arrangement.

Assume in the vector field an origin and a right-handed rectangular set of axes. Then at any point of the field the component of the field-vector parallel to any chosen radius-vector is

$$\mathbf{V}_r = \mathbf{V}_x \cos\alpha + \mathbf{V}_y \cos\beta + \mathbf{V}_z \cos\gamma. \tag{1}$$

Write the rate of change in this component, for the special case of leaving the origin along the particular radius-vector; it is

$$\left(\frac{d\mathbf{V}_r}{dr}\right)_o = \left(\frac{\partial \mathbf{V}_x}{\partial x}\right)_o \cos^2\alpha + \left(\frac{\partial \mathbf{V}_y}{\partial y}\right)_o \cos^2\beta + \left(\frac{\partial \mathbf{V}_z}{\partial z}\right)_o \cos^2\gamma + \left\{\begin{matrix}\text{terms containing}\\ \text{products of cosines}\end{matrix}\right\}, \tag{2}$$

the derivatives being all particular values at the origin. Form now the mean for all orientations of the radius-vector, which gives as a well-known result,

$$\left(\frac{d\mathbf{V}_r}{dr}\right)_o = \frac{1}{3}\left\{\left(\frac{\partial \mathbf{V}_x}{\partial x}\right)_o + \left(\frac{\partial \mathbf{V}_y}{\partial y}\right)_o + \left(\frac{\partial \mathbf{V}_z}{\partial z}\right)_o\right\}. \tag{3}$$

In a uniform field eq. (2) and consequently eq. (3) become zero identically. For the general case, therefore, a finite value of eq. (3) measures non-uniformity; and it does this in terms of divergence, whose known local value occurs in the second member. Divergence then is three times the mean rate of change in the radial component of the field-vector, for all directions of departure from a given point. Its invariant property is apparent in both members of eq. (3), since XYZ are assumed arbitrarily; and the proportional factor $(1/3)$ can be removed by giving a modified expression to this invariance:

$$\left(\frac{d\mathbf{V}_r}{dr}\right)_1 + \left(\frac{d\mathbf{V}_r}{dr}\right)_2 + \left(\frac{d\mathbf{V}_r}{dr}\right)_3 = \left(\frac{\partial \mathbf{V}_x}{\partial x}\right)_o + \left(\frac{\partial \mathbf{V}_y}{\partial y}\right)_o + \left(\frac{\partial \mathbf{V}_z}{\partial z}\right)_o. \tag{4}$$

The subscripts denote radius-vectors forming a shifting set of permanently rectangular axes, the expression of taking mean values being now omitted as superfluous. But the most significant feature of eq. (3) is the necessary occurrence of mean value in generalized connection with divergence. This occurs both here and in other relations, as will be shown later. The usual mean-value theorems credited to Boussinesq are stated for potential fields only; and though the extension to any steady vector-field is obvious enough, it seems worth while to take the step explicitly; especially when a parallel procedure, which is perhaps a little more novel, is found applicable to curl, as we now proceed to make plain.

For the component of field-vector perpendicular to a given radius-vector we have the expression, within the XY plane,

$$\mathbf{V}_P = \mathbf{V}_y \cos\alpha - \mathbf{V}_x \sin\alpha. \tag{5}$$

Then for all orientations round the chosen origin and lying in the XY plane we obtain, essentially by repeating the previous process, the mean value of the rate of change in this perpendicular component as we leave the origin,

$$\left(\overline{\frac{d\mathbf{V}_P}{dr}}\right)_o = \frac{1}{2}\left\{\left(\frac{\partial \mathbf{V}_y}{\partial x}\right)_o - \left(\frac{\partial \mathbf{V}_x}{\partial y}\right)_o\right\}(XY). \tag{6}$$

Similarly for the two other reference planes,

$$\left(\overline{\frac{d\mathbf{V}_P}{dr}}\right)_o = \frac{1}{2}\left\{\left(\frac{\partial \mathbf{V}_z}{\partial y}\right)_o - \left(\frac{\partial \mathbf{V}_y}{\partial z}\right)_o\right\}(YZ);$$
$$\left(\overline{\frac{d\mathbf{V}_P}{dr}}\right)_o = \frac{1}{2}\left\{\left(\frac{\partial \mathbf{V}_x}{\partial z}\right)_o - \left(\frac{\partial \mathbf{V}_z}{\partial x}\right)_o\right\}(ZX). \tag{7}$$

Here again the rates of change, and consequently the mean values that occur, become identically zero in a uniform field; while finite values for equations (6) and (7) may be used to measure this second aspect of non-uniformity. We have in fact introduced for each reference plane a quantity equally proportional to the curl-component normal to that plane; and the form of the equations exhibits the invariant nature of the quantities in the second members; since the axes XYZ are selected at random; and offers another definition of curl through the first members. With each reference plane we can associate curl whose local measure is twice the mean rate of change in the component of the field-vector perpendicular to the radius-vector, for all directions of departure in that plane, as we leave a given origin. One method of removing the proportional factor (1/2) is by using the statement of invariance, as was done in writing eq. (4). For any pair of permanently rectangular radius-vectors congruous with X and Y

$$\left(\frac{d\mathbf{V}_{P_1}}{dr_1}\right)_o + \left(\frac{d\mathbf{V}_P}{dr}\right)_o = \left(\frac{\partial \mathbf{V}_y}{\partial x}\right)_o - \left(\frac{\partial \mathbf{V}_x}{\partial y}\right)_o, \tag{8}$$

the difference of sign in the two members harmonizing with the usual conventions.

The results reached above do not introduce explicitly the idea that curl is of vector nature. But that link may be supplied by transferring consideration from the vector-component ($\mathbf{V}_P$) to its vector-moment $[\mathbf{rV}]_{xy}$ which is axial on Z. We write, for the XY plane,

$$\mathbf{M}(z) = [\mathbf{rV}]_{xy} = [\mathbf{rV}_P] = rV_P. \tag{9}$$

And for the rate of growth in proceeding along a given radius-vector,

$$\frac{d\mathbf{M}(z)}{dr} = [\mathbf{r}_1\mathbf{V}_P] + \left[\mathbf{r}\frac{d\mathbf{V}_P}{dr}\right] = V_P + r\frac{dV_P}{dr}, \tag{10}$$

where $\mathbf{r}_1$ is a unit vector, and the third members of the equations are meant to indicate that they can be used for calculation. This rate and its companions for the X and Y axes have not the general value zero for a uniform field; in which, for example, the third member of eq. (10) becomes V_P, constant along any particular radius, but changing value with orientation. The mean rate for all orientations, however, does vanish for each reference plane in a uniform field; and that fact yields a starting-point for measuring the departure in this respect of a given vector field from uniformity. For the general case dV_P/dr of eq. (10) has a finite value; and assuming linear relation near the limit ($r = 0$), the proportional factor depending on orientation,

$$V_P = (V_P)_o + r\frac{dV_P}{dr}, \tag{11}$$

which reduces eq. (10) to the form,

$$\frac{d\mathbf{M}(z)}{dr} = (V_P)_o + 2r\frac{dV_P}{dr}. \tag{12}$$

Next imagine the process of finding the mean rate of change of this vector-moment round Z expressed for a circumference of unit radius. The first term in the second member evidently contributes on the whole nothing to the mean. The result then reduces to

$$\left(\overline{\frac{d\mathbf{M}(z)}{dr}}\right) = 2\left(\overline{\frac{dV_P}{dr}}\right)_{r=1} = 2\left(\overline{\frac{dV_P}{dr}}\right)_o = \text{curl } (z). \tag{13}$$

Curl is measured for any axis, therefore, by a mean rate of change of vector-moment round that axis. The application at unit distance of the value of the derivative for the origin is based on the ordinary double aspect of a limiting rate.

The conditions supposed to lie behind the result of eq. (13) give an easy transition to the specification of curl as the limiting ratio of a line-integral to an area. Consider the particular case of a circle in the XY plane and centered on the origin. Assuming linear relation in preparation for the limit ($r = 0$), we have

$$\text{curl } (z) = \operatorname*{Lim}_{r=0}\left[\frac{\int_0^{2\pi}\left\{(\mathbf{V}_P)_o + r\frac{d\mathbf{V}_P}{dr}\right\}rd\gamma}{\frac{1}{2}\int_0^{2\pi}r^2d\gamma}\right] = \frac{2\int_0^{2\pi}\frac{d\mathbf{V}_P}{dr}\cdot d\gamma}{2\pi} = 2\left(\overline{\frac{d\mathbf{V}_P}{dr}}\right)_o; \tag{14}$$

the first term in the integral reducing to zero as before; and the ratio is independent of the radius. But if $r = 1$ the area $= \pi$; and we are thus made aware of a reason for the proportional factor of equations (6) and (7) of different type from that assigned previously. The means found for one complete revolution (2π) are half the quotient when the divisor is (π). The circumstances for divergence are so nearly parallel that it seems necessary only to indicate the result briefly. In terms of surface-integral and bounded volume we can write, for a sphere of any radius round the origin, and again with assumed linear relation,

$$\text{Div.} = \operatorname*{Lim}_{r=0}\left[\frac{\int_0^{4\pi r^2}\left\{(\mathbf{V}_r)_o + r\frac{d\mathbf{V}_r}{dr}\right\}dS}{\frac{4}{3}\pi r^3}\right] = 3\left(\overline{\frac{d\mathbf{V}_r}{dr}}\right)_o. \tag{15}$$

The third member here is seen to verify eq. (3). The radius of the sphere does not appear in the result; and from the special choice $r = 1$ we can trace the origin of the proportional factor to the simple fact that the volume $(4\pi/3)$ is then one third of the total solid angle (4π) for which the mean value was formed previously. And finally these interpretations of the connected processes furnish a plain reason for what is not otherwise so obvious: Why these limit-ratios for evaluating curl and divergence are independent of the *form* of the bounded surface and volume in the corresponding finite terms of the ratios. For any such approach toward the limit, the linear relation becomes a valid substitute at the vanishing-point; while curl and divergence involve in effect only the mean values of the respective *linear coefficients* at the origin.

The aspects of curl and divergence here developed seem to recommend themselves as offering a plan of straightforward description and measurement, applicable immediately to the vector distribution and arrangement, without the intervention of Ampère meshes, incompressible fluids, and the like; indispensable perhaps as historic scaffolding, but not perpetually to be retained after thought has clarified. We may note by way of conclusion one point about the factors $(1/2)$ and $(1/3)$. Their unconstrained occurrence seems to open a question of final choice between two standard expressions for curl, at least. It will doubtless always be felt as awkward that *twice* the angular velocity measures curl in cases of rotation, to mention only one item among several. On this issue the applications might prove to fall into two classes, each with its own best convenience. A group of such related matters is suggested—including the bearing of this point of view on vector potential—to which I propose to return in a second paper.

UNIVERSITY OF CALIFORNIA.

A COMPENSATING, LINEAR SCALE, HOT-WIRE AMMETER.

By A. H. Taylor.

THE objectional features of the usual hot-wire ammeters are too well known to deserve more than a brief enumeration. A shifting zero, a non-linear scale and a shunt ratio which depends on temperature, current strength and frequency seem to be the most obnoxious features of these instruments. At the same time they are valuable instruments for work with variable currents, especially where the frequency is very high.

In connection with the equipment of a radio-electric station at the University of North Dakota, the writer had need of an ammeter to put in the antennæ circuit, and not finding on the market a suitable instrument at a low price, he had one built by the departmental mechanician. This instrument has so satisfactorily met all objections, including the item of expense, that it is hoped that this brief description of it will be of value.

The wire used is No. 34 "Therlo," furnished by the Driver Harris Wire Co., and up to 0.6 ampere per strand seems to suffer very little elastic after effect when the heating current is removed. The sagging

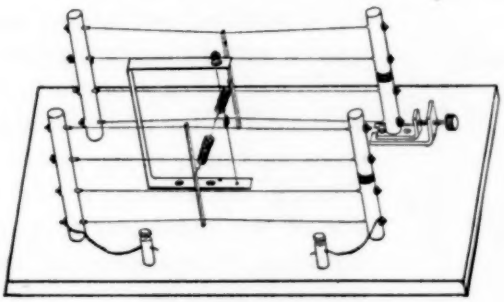

Fig. 1.

of a wire spanned between two supports is utilized to rotate a mirror or deflect a needle as the case may be. The instrument also has the differential feature, as has the Leeds and Northrup A.C.–D.C. comparator, but unlike the latter, the currents to be compared are carried through two widely separated and highly insulated circuits, enabling one to use the ammeter on high tension and high frequency circuits.

Fig. 1 shows one view of the instrument with the cover removed. There are two independent systems of four Therlo wires each, stretched between fiber posts set on an insulating base. Only two of these wires on either side take part in the indication of the instrument, the others being connected in at will as shunts, to alter the sensitiveness of the ammeter. These shunts carry practically the same currents as the indicating wires, hence have the same temperature, and thus give a shunt ratio independent of the current strength.

Owing to the small diameter of indicating and shunt wires there is but little skin effect. What little skin effect remains produces no effect on the shunt ratio, although it does undoubtedly slightly affect the sensibility of the ammeter as a whole. The two indicating wires on each side of the instrument are connected by light fiber bridges which pass inside of the shunt wires.

The right and left sides of the ammeter are made as symmetrical as possible and the initial tension on all wires can be adjusted by means of lock nuts on the terminal lugs which pass through the corner posts. After adjustment is once attained the lugs can be held fast by the set screws in these posts.

Referring to Fig. 2, one of the pair of active wires on each side is represented by W_1 and W_2; the vertical bridges by B_1 and B_2; the initial length of the wires by L; the expanded length by $L + \Delta L$; the needle by N; the mirror by M, placed on the suspension or pivot, and the corner posts by $P_1, \ldots P_4$. The last post, P_4, is mounted on a sliding base operated by a screw with an insulating head which passes outsid e of the case of the instrument. This movement of the post provides for a simultaneous slackening of the pair of wires W_2, thus allowing the sensitiveness of the right-hand side of the instrument to be altered, as the deflection, X, depends on the amount of initial tension in the wires.

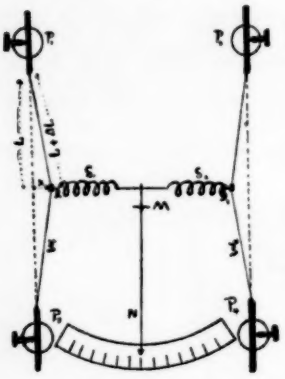

Fig. 2.

The differential feature is provided for by the two similar springs, S_1 and S_2, which connect the two bridges with each other and with the end of the needle.

Further insulation of the right from the left-hand side of the instrument is attained by separating the springs S by an inch of silk thread. With this arrangement only half of the deflection X is reproduced at the end of the needle, but this disadvantage is more than offset by the advantage

of having a zero which is not fluctuating with the temperature. The temperature within the case of the ammeter after continued use runs up some 20 or 30 degrees, but the zero shift is scarcely noticeable, owing to the compensating action of the two sides of the instrument.

The ammeter can be used as a direct reading instrument, or as a comparator when both sides have been adjusted to the same sensitiveness. In the latter case the unknown current is sent through one side and the deflection reduced to zero by passing a known current through the other side. The pivot, or rather the suspension at M, is a coarse silk thread, held very taut. The readings given here were taken with a mirror and a scale at 50 cm. distance.

The following is an approximate calculation of the value of the deflection X in terms of current, with the slightly erroneous assumption that the initial position of the indicating wires is in a straight line connecting the corner posts. The temperature of the wires is estimated not to exceed 400 degrees, so that the heat is gotten rid of mainly by convection and conduction, both of which would depend on the temperature difference between the wires and the surrounding space. Equilibrium is then expressed by the equation

$$I^2R = K(T - T_0). \tag{1}$$

From the geometry of Fig. 2 we see that

$$X^2 + L^2 = L^2 + 2L(\Delta L) + (\Delta L)^2, \tag{2}$$

or very approximately,

$$X^2 = 2L \cdot \Delta L.$$

Since

$$\Delta L = L(T - T_0)a, \tag{3}$$

where "a" is the expansion coefficient of the wire, we have from (1) and (3)

$$I^2R = \frac{K \cdot \Delta L}{aL}. \tag{4}$$

Finally, from (4) and (2)

$$I^2 = \frac{K}{2L^2 \cdot a} \cdot \frac{X^2}{R},$$

or, $I = CX$, where $C^2 = K/2aL^2R$, a constant provided that R and a are constant. Now for Therlo wire R is very nearly constant, but so far as I am aware a is undetermined.

A series of measurements showed that the average resistance per wire between 0.1 amp. and 0.6 amp. lay between 3.2 and 3.13 ohms, a variation of about 2 per cent.

We see then that for measurements not requiring too high a degree of precision the ammeter could be conveniently used with an external constant resistance multiplier on low frequency circuits.

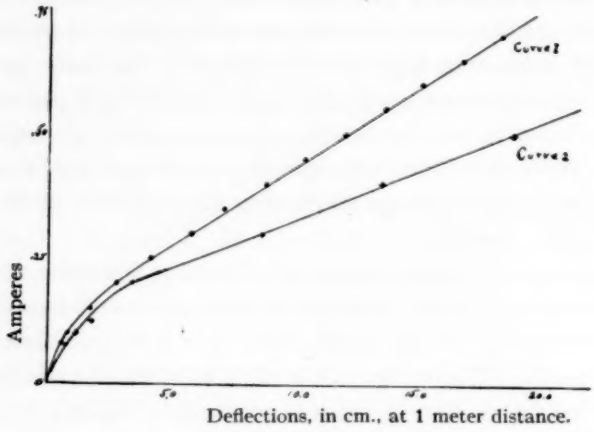

Fig. 3.

An inspection of curves 1 and 2 for two different tensions of the wires on the right-hand side of the instrument, will show not only how the sensibility is changed with the initial tension, but how the theoretical linear relationship between current and deflection is, after the first tenth of an ampere or so, almost exactly fulfilled. Curve 2 is for the higher tension.

The reason for the steeper rise of the first part of these curves is not apparent from the geometry of the instrument, but is rather to be deduced from the more rapid relaxation of the tension during the smaller expansions. It is also possible that the coefficient "a" is not constant, although this does not seem likely. This point is open to further investigation.

To calibrate the ammeter the maximum safe current is passed through both sides in series and the tension adjusted on the right-hand side until the deflection is zero. Then three sets of observations are taken: one for the right hand, one for the left-hand side, and one for the two, *i. e.*, a "difference" set. Table I. shows a set taken by a student in electrical measurements, curve 3 representing the results. A Weston mil-ammeter was used as a standard.

Curve 3 represents the data for both sides of the ammeter, right-hand side values being located by the dots, and left-hand by the crosses. For this test the two active wires on each side of the ammeter were connected in series. Placed in parallel, a far more efficient method of using the instrument, it would have one quarter the series resistance provided

TABLE I.

Right Side.		Left Side.	
Deflection.	Amperes.	Deflection.	Amperes.
.70 cm.	.082	.70 cm.	.079
.90	.090	.90	.091
1.10	.099	1.05	.100
1.25	.109	1.20	.109
1.50	.119	1.50	.121
1.85	.133	1.90	.139
2.30	.151	2.60	.160
3.40	.189	3.70	.199
5.40	.242	5.20	.229
8.70	.321	9.10	.327
16.40	.480	11.40	.378
21.40	.579	16.90	.489
23.60	.600	21.90	.584

TABLE II.

Left Side.	
Deflection.	Amperes.
.25 cm.	.10
1.05	.20
3.65	.40
7.25	.60
11.60	.82
16.40	1.02
21.20	1.20

that the two active wires were of the same length. In this case doubling the currents of Table I. should give the same deflections. Table II. gives the data for this connection, for one side of the ammeter. The circles above curve 3 represent these currents of Table II. divided by two. It is seen then that it takes a little more than twice the current to get the same deflection with this connection.

This is due to the fact that the two heating wires are not quite equal in length; thus their parallel resistance is somewhat less than one quarter of their series resistance, requiring therefore more than double the current to obtain the requisite heating effect. This error in construction will be easy to remedy.

Dividing the scale of the ammeter into equal parts, it is seen that for most practical purposes, neglecting the first 12 per cent. of the range, the calibration curve is sufficiently represented by the formula $D = mI + b$. Thus two constants m and b determine the calibration. The cost of construction, including labor, is about eight dollars.

UNIVERSITY OF NORTH DAKOTA.

APPLICATION OF THE SELENIUM CELL TO PHOTOMETRY.

BY A. H. PFUND.

THE remarkable property possessed by selenium of changing its electrical resistance upon illumination has led many investigators[1] to construct photometric devices involving the use of a selenium cell. In spite of all past efforts, however, the selenium cell has failed to establish itself in the field of exact photometry—a state of affairs directly attributable to the complex peculiarities displayed by selenium. Recently the writer was asked to devise a method of measuring the energy in the visible spectrum carried by a beam of monochromatic light which had suffered diffuse reflection from a plaster of paris surface. In view of the fact that bolometers, etc., are entirely too insensitive for work of this nature, the selenium cell was chosen. Not only is the latter tremendously sensitive, but, in addition, its range of sensitiveness extends through the entire visible spectrum. It is the object of this paper to define the conditions under which accurate results may be obtained and also to call attention to the difficulties which present themselves when the selenium cell is applied to heterochromatic photometry.

GENERAL OUTLINE OF THE WORK.

It is first essential to establish a sensibility curve, *i. e.*, a curve plotted between changes in the resistance of the selenium cell and the wavelengths of the exciting light. In all cases, the energy carried by the exciting beam of monochromatic light is kept constant. With a sensibility curve at hand, the method of measuring energy is obvious. If the energy carried by an intense beam of monochromatic light is to be measured, the intensity is cut down by a *known* amount by means of a rotating-sector or some other geometrical device until the galvanometer deflection is the same as that indicated on the sensibility curve; if, conversely, a faint beam of known energy content is desired, again a fairly intense beam is cut down to the desired amount—only in this case the rotating sector is kept running instead of being removed, as in the previous case. This course of procedure is by far the safest as the relation between galvanometer deflections and energy has been found to change in a com-

[1] See "Die Elektrischen Eigenschaften u. die Bedeutung des Selens in der Elektrotechnik," Dr. Chr. Ries, 1908.

plicated manner as one passes through the spectrum. It has therefore seemed most advisable to refer all measurements to the initial sensibility curve and thus keep clear of the difficulties which are unavoidable in any other course of procedure. Next, the relation between the incident energy and the resulting change in resistance of the selenium cell is investigated and a law connecting these two quantities is established. The important result follows that, under the adopted method of procedure, the sensibility curve is not fixed but varies in a remarkable manner with the intensity of illumination. Finally, the applicability of Talbot's law, and hence the justifiability of using a rotating sector is established.

Apparatus and Method of Measurement.

The light from a Nernst lamp L (Fig. 1), after having been passed through the mirror spectrometer $S_1M_1PM_2$, was resolved into a spectrum

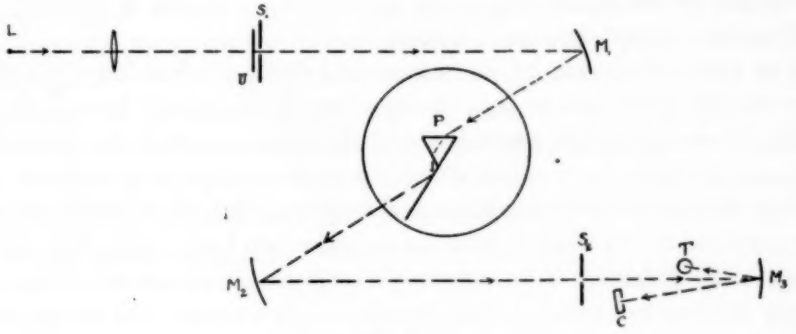

Fig. 1.

which was brought to a focus in the plane of the slit S_2. This slit transmitted a bundle of quasi-homogeneous rays which fell upon the cancave mirror M_3 and, after reflection, were projected either upon the thermal couple T or the selenium cell C. The latter was connected in the circuit of a simple potentiometer—as shown in Fig. 2. The object of the auxiliary circuit $E_1R_1R_2$ was to introduce a slight counter-E.M.F. at the galvanometer terminals and thus annul the initial "dark" deflection. It has been pointed out in one of my previous papers[1] that a selenium cell will furnish reliable results only when galvanometer deflections resulting from comparatively short exposures are taken. The device for procuring such exposures consisted of a circular disc of heavy paper from which a 75° sector had been cut out and which was mounted on the prolongation of the "second-hand" axis of a small alarm clock. This arrangement, which yielded exposures of 12.5 seconds, was mounted at U

[1] A. H. Pfund, Phil. Mag., Jan., 1904.

in such a manner that the axis of rotation of the paper disc passed through the prolongation of the slit and was perpendicular to the plane of the slit. Under these conditions the light was admitted and cut off abruptly and results agreeing within less than 2 per cent. were obtained.

Two Giltay cells were studied and each exposed an area of about 1 square mc. and had a resistance of 3×10^6 ohms. Their behavior was so nearly the same that the results obtained from but one will be presented. Both were extremely sensitive and, under the comparatively faint illumination to which they were exposed, showed no observable hysteresis due to successive exposures to differently colored beams. Following in part the course of procedure laid down by Stebbins,[1] the full current was permitted to pass through the cell for several hours before a series of measurements was taken and the temperature of the room was brought to about 21° C. after which it was kept constant to within 1° C. When, finally, a steady condition had been established, the selenium cell was exposed to the white light of an incandescent lamp until about twice the average deflection to be expected in later measurements was obtained. After waiting 3–4 minutes, this procedure was repeated and after a subsequent interval of 4 minutes the resistances in box R_1 were readjusted in order to bring the galvanometer back to zero. All this means that the "dark" resistance, to which all subsequent measurements are referred, is not that acquired by the cell after having been in darkness for a long time, but that acquired by the cell subsequent to an exposure to white light. Such a course of procedure was found necessary for the reason that a cell which had been kept in darkness for a long time and which was then exposed to light of the intensity used, required more than 10 minutes to return to its initial "dark" resistance—an interval of time entirely too long to be practicable. Under the actual conditions of the experiment the time interval rarely exceeded 4 minutes.

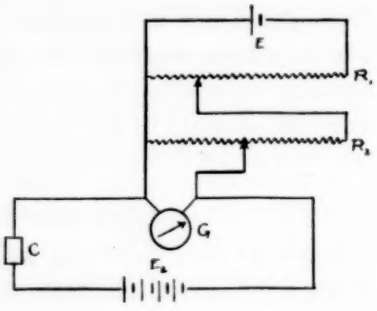

Fig. 2.

Both resistance boxes R_1 and R_2 contained 10,000 ohms. The D'Arsonval galvanometer G had a resistance of 469 ohms, a working sensibility of 280 megohms and a period such that 8.0 seconds were required to reach maximum deflection from rest. The other D'Arsonval galvanometer, which was connected to the thermopile, had but 7 ohms resistance

[1] J. Stebbins, Astrophys. Jnl., XXXII., p. 185, 1911.

and a working sensibility of 2×10^{-9} amperes. The two-junction-vacuum thermopile T as well as the charcoal evacuator (used to produce and maintain high vacua) are of new design and have already been described elsewhere.[1] The sensibility was such that the total radiation from a meter-candle falling upon the blackened junctions (of 2 sq. mm. area) through a rock-salt window, gave rise to a deflection of 500 mm. In the present experiments, the thermal junctions were protected by a glass window—a similar window having been placed over the selenium cell in order to compensate for any selective absorption of the glass.

Sensibility Curve.

Having already defined the conditions under which sensibility curves are obtained, it only remains to present a typical curve obtained under very faint illumination. In order to obtain deflections of sufficient magnitude, a storage-battery of 20 volts was inserted at E_2 (Fig. 2). Successive galvanometer deflections resulting from exposures of the selenium cell under identical conditions, rarely differed by more than 1 per cent. In the following table λ is the wave-length of light used; E is the energy as measured by the deflections of the galvanometer connected to the thermal-junctions; D_0 is the deflection of the galvanometer connected to the selenuim cell and D_c is the deflection corrected for the constant energy of 22.0 mm. in column E.

Table I.

λ $\mu\mu$	E Mm.	D_0 Mm.	D_c
453	22.5	109.5	108.2
494	22.0	113.5	113.5
521	22.0	118.0	118.0
556	21.2	119.9	122.0
601	22.2	118.0	117.5
625	22.0	107.4	107.4
651	21.7	86.1	87.0
663	22.0	80.0	80.0
675	21.6	75.8	78.0
686	21.6	75.0	77.5
711	22.5	58.0	56.2
738	21.3	35.0	37.5
785	22.0	5.0	5.0

These results are presented graphically in Fig. 3. It may be added that it was found possible, after or during a series of measurements, to

[1] Phys. Review, April, 1912. Abstract of paper presented at Washington meeting of Am. Phys. Soc., Dec., 1911.

expose the cell to light of any desired color and to repeat previously obtained observations very closely. These results establish, I think, the fact that the selenium cell may be used with success to measure the intensity of monachromatic light accurately

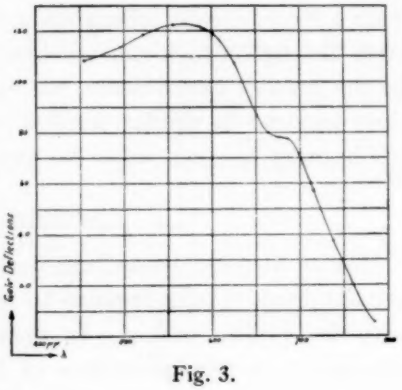

Fig. 3.

Talbot's Law.

In view of the proposed use of a rotating-sector, a careful study of the applicability of Talbot's law to the selenium cell seemed imperative. No matter what type of instrument is used to measure radiant energy, the use of a rotating sector must first be justified,—for it is by no means evident that intermittent flashes of intense light will produce the same effect as a continuous beam of the same total energy content. In order to put these matters to test, the following course of procedure was followed out: a rotating sector containing angular openings of 180°, 60° and 20° was so mounted in front of the slit of the spectrometer, that any one of the three apertures could be used at will. First the thermal-junction was tested by means of the rotating-sector and it was found that over the entire range of the 50 cm. scale, the deflections were rigorously proportional to the energy. Then the work on the selenium cell was begun and it may be noted that the cell was exposed always for 12.5 seconds. The cell was first exposed to the uninterrupted beam and the resulting galvanometer deflection was noted—likewise, the energy carried by this beam was measured; then the 180° portion of the sector was made effective, the selenium cell was again exposed and the galvanometer deflection was noted. Next, the rotating sector was removed and the beam, which was concentrated on the thermocouple, was cut down by means of an optical wedge to 50 per cent.—as measured by the deflections of the galvanometer connected to the thermal-couple; again the selenium cell was inserted. If, now, the deflections due to the selenium cell, excited by a continuous beam reduced to 50 per cent. of its initial intensity, is the same as the deflection obtained when the 180° sector was used, then Talbot's law holds. The long table of results bearing on this question is omitted. Suffice it to say that Talbot's law holds to within 2 per cent. (the limit of accurary) through the spectral region $\lambda = 450$–$785\mu\mu$ for an intensity range of 1 : 18.

The sector was of the usual symmetrical type having equal angular openings on opposite sides of the center of rotation. Usually the disc was driven at high speed—about 30 revolutions per second, *i. e.*, 60 interruptions. It was found, however, that the "inertia" of the galvanometer as well as that of the selenium cell made it permissible to use so low a frequency of interruption (about 10 per sec.) that violent flicker was perceptible to the eye. These facts are brought out in the following table where galvanometer deflections for different frequencies of interruption are recorded.

TABLE II.

λ $\mu\mu$	60 Interruptions per Second. Mm.	10 Interruptions per Second. Mm.
494	85.1	85.0
711	145.0	145.8
750	116.5	115.9

LAW CONNECTING E AND D.

A number of investigators[1] have formulated laws connecting the intensity or energy E of the exciting light and the resulting galvanometer deflection D which is a measure of the change in conductivity of the selenium cell. The variety of results obtained would seem to indicate that the experimental conditions in the several cases must have differed widely. As the final law will differ with the character of the cell, the time of exposure and the absolute intensity of light, it is to be stated definitely that the cell studied in this investigation was made by Giltay, the time of exposure was 12.5 seconds and the mean intensity of light corresponding to E = 60 mm. was 2.1 lux (the last result was obtained by balancing, photometrically, the total radiation from a Hefner lamp against the monachromatic radiation λ = 590 $\mu\mu$). Due to experimental difficulties, no attempt was made to establish a law for very intense and very faint illuminations. From an inspection of a table of preliminary results it was evident that the law connecting E and D is very approximately of the form:

$$D = KE^B,$$

where K and B are constant as long as the wave-length of the exciting light remains unchanged. To evaulate B it issimply necessary to note the deflections D_1 and D_2 corresponding to the energies E_1 and E_2. Thus, from the above relation it is found that

$$D_1 = KE_1^B, \quad D_2 = KE_2^B,$$

[1] See Ries, l. c.

whence

$$B = \frac{\log \frac{D_1}{D_2}}{\log \frac{E_1}{E_2}}.$$

To give some idea as to the constancy of B the following data (Table III.), taken from a preliminary series of results, are presented. Equally consistent results were obtained throughout the spectrum.

TABLE III.

Wave-length 460μμ.

E Mm.	D Mm.	B
100.0	254.0	
74.1	220.0	0.515
42.5	164.6	.501
21.0	115.1	.508
7.9	68.0	.518
5.6	58.3	.508
		Average 0.510

All calculations of B are referred to the highest values of E and D.

To show how the value of B varies with change of wave-length a separate series of measurements was carried out. The results are given in Table IV., where E_0 is the energy of the beam and D_0, the galvanometer deflection resulting from the increased conductivity of the selenium cell; D_1 is the deflection obtained upon cutting the light down to 50 per cent. of its initial intensity by means of the rotating sector and B is the value of the exponent.

TABLE IV.

λ μμ	E_0 Mm.	D_0	D_1	B
450	20.0	88.5	62.2	0.510
494	38.0	125.0	87.0	0.513
556	61.5	167.0	115.5	0.525
601	59.8	160.0	108.0	0.535
651	59.2	117.0	79.0	0.544
666	58.0	120.0	80.2	0.582
681	58.2	154.0	93.5	0.712
696	61.4	202.0	109.0	0.877
711	59.5	194.5	97.3	1.00
745	82.0	84.2	41.9	1.01
785	164.0	30.6	15.2	1.01
830	180.0	9.0	4.5	1.00

It will be noted that for wave-lengths shorter than 650$\mu\mu$ the deflections are approximately proportional to the square-root of the energy ($B = 0.5$) while for wave-lengths greater than $\lambda = 700\mu\mu$ the deflections are directly proportional to the energy ($B = 1.0$).

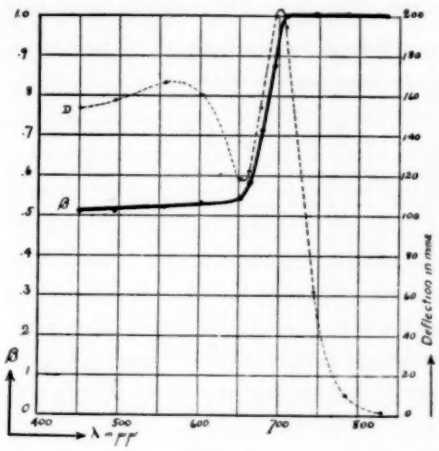

Fig. 4.

The results are presented graphically in Fig. 4 where the heavy curve represents the various values of B and the dotted curve, the sensibility curve corresponding to $E = 60$ mm.

These results offer, I think, a clue as to the difference between the so-called "hard" and" soft" Ruhmer cells. They are characterized by the fact that the former are compararively sensitive to variations of intense sources of light and comparatively insensitive to faint sources, while the latter behave in the opposite manner. Ruhmer, it appears, has developed a method of sensitizing cells so that the position of the sensibility maximum is under control. Let it be assumed that the law between E and D, previously deduced, also applies to Ruhmer cells and that we have two cells whose maxima lie, respectively, in the green and red. Let these two cells be placed side by side and exposed to the same source of white light at a distance of 1 meter and let the respective circuits be so adjusted that the galvanometer deflections, resulting from an exposure of the cells to light, are the same. If now the source be removed to a distance of 3 meters the intensity will drop to one ninth of its original value and, as a result, the deflections due to the "red" cell will likewise decrease to one ninth of its original value, while the deflection of the "green" cell will decrease only to about one third, since its maximum lies in that region of the spectrum in which the square-root law applies. Thus, under this weak illumination, the "green" cell is approximately three times as sensitive as the "red." If, on the other hand, the source be approached to within one third of a meter, the "red" cell will be found to be approximately three times as sensitive as the "green." Thus the "red" or "hard" cell is relatively the more sensitive to intense illumination, while the "green" or "soft" cell is relatively the more sensitive under faint illumination. Cells of these two types are not at my disposal. It would be most interesting to put the above deductions to an experimental text.

CHANGE OF SENSIBILITY CURVE WITH INTENSITY OF ILLUMINATION.

Since it has been demonstrated that the exponent B is not constant throughout the spectrum, it is evident that the form of the sensibility curve must vary with the intensity of illumination. To show how marked these variations are, the curves, shown in Fig. 5 have been pre-

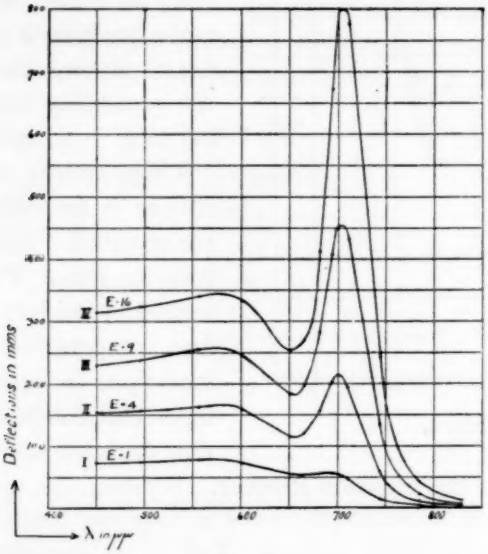

Fig. 5.

pared. The intensities or energies corresponding to Curves I., II., III. and IV. are in the ratios 1:4:9:16—the Curve II., which has already been presented in Fig. 4, serving as a basis for calculation. This procedure is entirely permissible since B has been found to remain sensibly constant over an intensity range even greater than 1 : 16. As a matter of fact, curves of all the types shown have been actually found and the deductions, based on this work, are fully borne out by the curve already presented in Fig. 3 (obtained under very feeble illumination), as well as by the sensibility curve presented in my earlier papers.[1] Referring once more to Fig. 5, it is to be observed that Curve I., obtained under the faintest illumination, has its maximum in the greenish-yellow and indicates but faintly the existence of a maximum in the red. As the energy is increased 4 times, Curve II. is obtained and from this it is seen that, while the deflections in the region of shorter wave-lengths increase only about two times, those in the red increase four times. The explanation of the remaining curves and the reason for the tremendous increase of the maximum near $700\mu\mu$ follows in a similar manner.

[1] A. H. Pfund, Phil. Mag., Jan., 1904; Phys. Zeitschrift., 10, p. 341; PHYS. REVIEW, May, 1909.

With these results as a basis, I venture the attempt to reconcile the discordant results obtained by various observers concerning the position of maximum sensibility in the spectrum. While[1] Minchin, Mercadier and Adams found that the greenish-yellow and yellow rays produced the greatest effect, Sale, Siemens, Forsmann, Marc, Pfund, Schrott and Athanasiadis found that the red rays were most effective. The curves in Fig. 5 would seem to indicate that both contentions may be correct—depending on the intensity of illumination. Under very faint illumination the greenish-yellow rays are most effective, while under very intense illumination the red rays produce by far the greatest effect. It is to be distinctly understood that these deductions apply only under the condition that the illumination of the cell takes place for a comparatively short interval of time—entirely insufficient to permit the cell to assume a steady resistance under continued illumination. As to whether or not relations, similar to those here presented, also obtain when the cell is exposed until an equilibrium condition has been established, is immaterial for the present, since such a procedure would exclude the practical applications of the selenium cell to photometric measurements.

Referring once more to Fig. 5, it is evident that very great caution must be exercised in using a selenium cell to measure the intensity of white (undispersed) light. Assuming that the selenium integrates the effects of the various colors according to the sensibility curves presented, it is clear that the galvanometer deflections obtained will vary neither directly as the energy or its square-root but according to some power determined jointly by the values of B (Fig. 4) and by the distribution of energy in the spectrum of the source. Without showing in detail how erroneous results might be obtained, it will suffice to say that only that method of procedure is free from objections which involves a calibration of the selenium cell against the same source whose variations in intensity are to be measured subsequently.

Hysteresis Effects and Constancy of Behavior.

A discussion of the reliability of the results obtained by means of the selenium cell and of its constancy of behavior with time is, naturally, of vital importance in this connection. While undoubtedly hysteresis, *i. e.*, the effect due to previous exposure, is noticeable when the illumination is strong, no such effect was ever observed in the previous experiments—all of which were carried out under faint and only moderate intensities. In fact, while a sensibility curve was being determined it was found that any reading could be repeated no matter to what colored light the cell had last been exposed. As for constancy of behavior with time it may

[1] See Ries, l. c.

be stated that while every well-made cell experiences an increase in resistance after having been made, it eventually reaches a steady state. While my experience with the Giltay cells does not cover a sufficiently great interval of time to make the drawing of conclusions permissible, I have found that cells made and tested by me in 1904 had retained all of their characteristics in 1909 when they were last tested. These cells had carbon electrodes and were preserved in an evacuated tube. It is hence to be concluded that a seasoned and well-made selenium cell is a reliable instrument.

SUMMARY.

The results obtained may be summarized as follows:

1. It has been established that the selenium cell, when used as a photometer, will yield accurate results if the following conditions are fulfilled:

(*a*) Monachromatic light must be used.

(*b*) An accurate sensibility curve must have been established.

(*c*) Exposures to light must be made automatically and must be of short duration.

2. Talbot's law has been found to hold (within the limits of accuracy of measurement) throughout the visible spectrum. The range of frequencies covered was from 10 to 60 interruptions per second.

3. For moderate intensities of illumination and for a range of intensities 1 : 18, the relation between the incident energy (E) and the galvanometer deflection (P) has been found to be represented by the expression

$$D = K \cdot E^B.$$

As a consequence of determining the various values assumed by B, it is found that from $\lambda 450$–$650\mu\mu$ the deflections are proportional, approximately, to the square root of the energy, while from $\lambda 700$–$830\mu\mu$ the deflections are directly proportional to the energy. An attempt is made to account for the behavior of "hard" and "soft" cells.

4. As a consequence of the variations of B with λ it is shown that the form of the sensibility curve varies with the absolute intensity of illumination. It is furthermore shown that, under very faint illumination, the selenium cell is most sensitive toward yellowish-green light, while, under intense illumination, the cell is by far the most sensitive toward red light. An explanation of the cause of disagreement between various investigators in their determination of the position of maximum sensibility is attempted.

5. The conditions are defined under which the selenium cell may be safely used to measure the intensity of white light.

JOHNS HOPKINS UNIVERSITY,
February, 1912.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-SECOND MEETING.

THE spring meeting of the Physical Society was held in the Jefferson Physical Laboratory, Harvard University, Cambridge, on Saturday, April 27, 1912, with President W. F. Magie in the chair.

The following papers were presented:

The Optical Properties of Crystalline Spheres formed by Precipitation in Gelatine. H. W. MORSE.

Some Further Results of Precision Measurements of Refractive Indices as a Function of Temperature. (By title.) F. A. MOLBY.

Absolute Formulæ for the Attraction of two Coaxial Solenoids. (By title.) GEO. O. OLSHAUSEN.

The Calculation of the Maximum Value of the Force between Two Coaxial Circular Currents. (By title.) FREDERICK W. GROVER.

Absorption and Reflection Spectra of Metallic Sulphides. A. TROWBRIDGE.

Distortions in Spectral Luminosity Curves Produced by Variations in the Character of the Comparison Standard and in the Surroundings of the Photometric Field. HERBERT E. IVES.

The Electron Theory of Magnetism. D. F. COMSTOCK.

Direct Measurement of the Velocity of Kathode Rays and the Variation of Mass with Velocity. E. LEON CHAFFEE.

The Effect of a Magnetic Field on Photo-electric Emission. A. W. HULL.

The Kinetic Energy of the Electrons emitted by Photoelectric Action. O. W. RICHARDSON and KARL T. COMPTON.

The Law of Fall of a Drop through Air at Reduced Pressures and a Redetermination of e. R. A. MILLIKAN.

A Theoretical Correction to the Drop Method of Determining e. ARTHUR C. LUNN.

The Characteristics of Vowels. D. C. MILLER.

Changes in the Dielectric Constant Produced by Strain. E. P. ADAMS and C. W. HEAPS.

On Mechanical Force from the Magnetic Field of a Displacement Current. R. H. GODDARD.

Rotations in the Metallic Arc. W. G. CADY.

Note on the Variation from Lambert's Cosine Law of the Emission from Tungsten and Carbon at Glowing Temperatures. A. G. WORTHING.

The Chaffee Gap as Interrupter for an Induction Coil. Demonstration. G. W. PIERCE.

Anomalous Magnetization in Iron. B. O. PIERCE.

Magnetic Induction in a Group of Oblate Spheroids of Iron. (By title.) S. R. WILLIAMS.

The Relations between the Optical and Electrical Constants of Metals. L. P. WHEELER.

The Diurnal Range of Temperature and Other Meteorological Elements at Different Levels above Mount Weather. WM. R. BLAIR.

The Thermal Properties of Liquid Water up to 80° and 12,000 Kgm. per Sq. Cm. P. W. BRIDGMAN.

Progress on a Redetermination of the Specific Heat of Steam. (By title.) A. R. NOTTINGHAM.

A Lecture Experiment to Show Variation of Elasticity with Temperature (Illustrated). A. TROWBRIDGE.

Variation of Electrical Resistance with Temperature. Oxides. A. A. SOMERVILLE.

The Electric Spark. Control of the Condenser Discharge by Ultra-violet Radiations. WILLIAM O. SAWTELLE.

The Relation between Current, Potential Difference and the Condition of the Electrodes, in the Case of Surface Ionization in a Vacuum. O. W. RICHARDSON and CHARLES SHEARD.

Heats of Dilution. W. F. MAGIE.

The Relation of Osmotic Pressure to Temperature. W. F. MAGIE.

The Maximum Intensity of Magnetization in Iron. (By title.) B. O. PIERCE.

Oscillographic Study of Current Charges Produced by a Carbon Transmitter in Wire and Wireless Telegraphy. G. W. PIERCE and E. L. CHAFFEE.

The Linear Expansion of Invar Steel between 100° C. and −190° C. (By title.) F. A. MOLBY.

An Attempt to use Centrifugal Separation on Liquid Air. (By title.) H. L. HOWES.

The Effect of Light on the Helix of Cathode Roys from a Wehnelt Cathode. (By title.) C. P. KNIPP.

Multiple Reflection of Short Electric Waves from Screens of Tinfoil Resonators. W. L. SEVERINGHAUS and W. S. NELMS.

An Accurate Potentiometer Method for Measuring Resistance. W. P. WHITE.

The Addition of Luminosities of Different Color. H. E. IVES.

Outline of a New Method for the Determination of e/m in Metals. D. F. COMSTOCK.

Adjourned at 4:30 P.M.

ERNEST MERRITT,
Secretary.

The Electron Theory of Conductors.[1]

By O. W. Richardson.

THIS paper is a further development of the matter of previous papers communicated to the Society. (For Abstracts see Phys. Rev., Jan. and Feb., 1912.) It is divided into two parts.

The first investigates some consequences of the hypothesis that the collisions of the electrons inside a conductor occur with centers of force which attract them with a force varying inversely as the cube of the mutual distance. Those electrons which contribute to the integrals which determine the thermoelectric properties can be divided into two groups, according as their orbits are closed or open. The additional hypothesis is introduced that the electrons whose orbits are closed do not contribute anything to the current of conduction. They may, of course, contribute to the polarization current.

The criterion which determines whether the orbit of an electron at any point is an open one, is that the kinetic energy should exceed the work of the attracting forces from infinity to the point in question. From this condition the proportion of the electrons, instantaneously present at a point where their potential energy is $\omega = -\varphi$, which are engaged in carrying the current, can readily be calculated. It is found to be

$$\chi\left(\frac{\varphi}{R\theta}\right) = \frac{2}{\sqrt{\pi}}\left[\sqrt{\frac{\varphi}{R\theta}}\,e^{-\frac{\phi}{R\theta}} + \int_{\left(\frac{\phi}{R\theta}\right)^{\frac{1}{2}}}^{\infty} e^{-x^2}dx\right].^2$$

This calculation assumes that the distribution of velocity accords with Maxwell's law and is independent of the distribution of potential energy, in agreement with the results obtained previously.

It has been shown in a previous paper that ν, the number of electrons in unit volume at a place where the potential energy is ω, is

$$\nu = \frac{ne^{-\frac{\omega}{R\theta}}}{\int_\tau e^{-\frac{\omega}{R\theta}}d\tau} = \kappa e^{-\omega/R\theta},$$

where n is the total number of electrons which can become free in the total volume τ, and κ is a function of θ only. The number of current-carrying electrons is therefore

$$\int_\tau \nu\chi\left(\frac{\varphi}{R\theta}\right)d\tau = \frac{2n}{\sqrt{\pi}}\,\frac{\int_\tau\left[\sqrt{\frac{\varphi}{R\theta}} + e^{\frac{\phi}{R\theta}}\int_{\sqrt{\frac{\phi}{R\theta}}}^{\infty} e^{-x^2}dx\right]d\tau}{\int_\tau e^{\frac{\phi}{R\theta}}d\tau}.^2$$

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

[2] These equations were affected by an error of calculation as originally given. They were changed to the above expressions in reading the proof April 20, 1912.

These formulæ lead to the right kind of variation with temperature in the case of poor conductors to agree with the results of experiment; but it is impossible to institute a quantitative comparison as there appears to be no independent method of determining the value of the integrals involved.

From this point of view the change of metallic resistance caused by a magnetic field might be caused very largely by the effect of the magnetic field in altering the proportion of open to closed orbits and would therefore not necessarily exhibit any close correlation with the magnitude or sign of the Hall effect.

The second part of the paper deals with the theory of the photoelectric effect. It was shown previously that if $N(\nu)$ is the number of electrons emitted when unit energy of frequency between ν and $\nu + d\nu$, and otherwise having the characteristics of the complete ethereal (black-body) radiation is incident on a body, at temperature θ, which only emits electrons under the influence of photoelectric action, then

$$\frac{c}{4}\int_0^\infty N(\nu)\,E(\nu)d\nu = A\theta^{\frac{1}{2}}e^{\int \frac{\omega}{R\theta^2}d\theta}, \qquad (1)$$

where c is the velocity of light. $E(\nu)$ is the complete radiation formula at temperature θ, A is a constant which is different for different substances but does not involve θ, ω is the heat absorbed when one electron is allowed to escape from the substance without performing external work and R is the gas constant for a single molecule. Now it appears that a thermodynamic investigation of the specific heat of electricity σ leads to the conclusion that

$$\frac{\partial\omega}{\partial\theta} = \tfrac{3}{2}R - e\sigma$$

and it may be shown that for all known substances $e\sigma$ is much smaller than $3R/2$. If we may disregard $e\sigma$ on account of its smallness we can put

$$\omega = \omega_0 + \theta\frac{\partial\omega}{\partial\theta} = \omega_0 + \frac{3}{2}R\theta,$$

where ω_0 does not involve θ and the right hand side of (1) then reduces to

$$A\theta^2 e^{-\frac{\omega_0}{R\theta}}.$$

Assuming as a sufficient approximation that $N(\nu)$ is independent of θ we may expand it in powers of ν^{-1}. Substituting Wien's formula

$$E(\nu)d\nu = \nu^3\varphi\left(\frac{\nu}{\theta}\right)d\nu$$

and expanding the exponential on the right, one finds, after equating coeffi-

cients of powers of θ, that

$$N_\nu = \frac{4}{c}\frac{A}{\nu^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\int_0^\infty \varphi(x)x^{1-n}dx}\left(\frac{\omega_0}{R\nu}\right)^n. \qquad (2)$$

The integrals in the denominators of (2) are not, however, mere numbers. According to the radiation formulæ of Planck and Wien they contain a parameter h/R, where h is Planck's constant. By changing the variable to $z = (h/R)x$ one finds

$$N\nu = \frac{c^2A}{2\pi}\frac{h}{R^2\nu^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\int_0^\infty z^{-n+1}x(z)dz}\left(\frac{\omega_0}{\nu h}\right)^n, \qquad (3)$$

where the integrals are now pure numbers. For example if Planck's formula is used $x(z) = (e^z - 1)^{-1}$. Hence

$$N_\nu = \frac{A}{\nu^2}\varphi\left(\frac{\omega_0}{\nu h}\right),$$

where $\varphi(\omega_0/\nu h)$ is a function of $\omega_0/\nu h$ *only*. Thus the curves giving the relative emission of electrons under the influence of isotropic radiation of varying frequencies may be constructed for all substances when the curve for any one substance and the values of ω_0 have been determined. Also the maximum and minimum values of $N\nu$ will be given by particular values of $\omega_0/\nu h$, the same for all substances.

In a rather similar manner it may be shown that the mean kinetic energy T_ν of the electrons emitted by light of frequency ν is of the form $\nu\Phi(\omega_0/h\nu)$ where Φ is a function of $\omega_0/h\nu$ only. Φ is different from φ but is the same for all substances.

These conclusions cannot be said to be established with any degree of mathematical rigor. It might be objected that the physical terms which have been neglected would, if included, possibly entirely change the nature of the solutions. I do not, however feel that that is the case. On the other hand the properties of the integrals which enter into (2) and (3) might cause the series to become divergent or non-existent. Using Wien's radiation formula as a sufficient approximation, the integrals in (3) become Γ functions, being in fact $\Gamma(2)$, $\Gamma(1)$, $\Gamma(0)$, $\Gamma(-1)$, etc., in succession. Thus the series disappears except for the first two terms. However, this result is entirely dependent on the fact that the index of the power of θ on the right-hand side of the integral equation is an integer. This cannot be considered to be necessary on theoretical grounds and empirical reasons might be brought forward in favor of substituting $\theta^{\frac{1}{2}}$ for θ^2. In that case the Γ functions are no longer infinite and it can be shown that the series are convergent when $\omega_0/h\nu$ is less than unity. If ω_0 has the values calculated from thermionic measurements this fraction is less than unity for such frequencies as give rise to photoelectric

emission. The fraction is also of the order unity. The condition

$$\frac{h\nu}{\omega_0} \geqq 1$$

is also required for emission to occur on Einstein's theory.

So far as I am able to judge the experimental data hitherto accumulated do not conflict with the results of these calculations. More direct tests are being carried out at present.

PALMER LABORATORY,
PRINCETON, N. J.

THE IONS FROM HOT SALTS.[1]

BY O. W. RICHARDSON.

THE first part of this paper deals with further measurements of the specific charge e/m of the ions from salts. The principal object of the investigation has been the determination of the nature of the negative ions which are emitted at low temperatures. Experiments have been made on CdI_2, BaI_2, SrI_2, CaI_2, Fe_2Cl_6, and $MnCl_2$. The negative thermionic currents from the iodides of the alkaline earth metals are surprisingly large, and apparently larger at low temperatures than those given by the corresponding oxides. Thus a layer of barium iodide only a few square millimeters in area was found to give a negative emission of about two microamperes before it was visibly hot. Heavy negative ions from all these substances have been detected, but as a rule they are mixed with an excess of electrons.

On account of various difficulties satisfactory measurements of e/m for the heavy ions have so far only been obtained in the cases of CaI_2, SrI_2 and BaI_2. In the case of all these substances the electric atomic weight of the heavy ions is in good agreement with the view that the ions are atoms of iodine carrying a single electronic charge (or, of course, molecules carrying two charges). The proportion of heavy ions to electrons is greatest when the salts are first heated and falls off with lapse of time. The proportion of heavy ions to electrons also diminishes as the temperature is raised, independently of the time effect.

It seems probable that in the case of these compounds the heavy ions are primary ions and are not formed by the combination of the atoms or molecules of the gas with electrons, since in the case of calcium iodide, freshly heated at a low temperature, all the ions had an electric atomic weight corresponding to I_- and no electrons could be detected. If this is so it would seem to be the only case known in which a negatively charged atom occurs as a primary gaseous ion.

Generally speaking the large negative emission which has been observed is not accompanied by any appreciable positive emission. In the case of SrI_2,

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

in order to measure e/m for the positive ions, it was found necessary to heat the salt to a very much higher temperature. The value of the electric atomic weight observed agreed very well with Sr_+ and is therefore in accord with Davisson's measurements with other salts of this metal.

Various time changes have been observed which are being investigated further.

It is interesting to observe that the salts which most readily form heavy negative ions are those containing the extremely electronegative element iodine, whilst those which most readily form positive ions are those of the extremely electropositive heavy alkali metals.

The second part of the paper is devoted to a brief discussion of the time changes which the ionization from heated salts exhibits. It was first shown in the case of the haloid salts of zinc, by Garrett and Willows, that the ionization at constant temperature increased to a maximum and subsequently diminished. The effects could be explained on the hypothesis that two successive decompositions took place, the first being unaccompanied by ionization and the second resulting in the emission of ions. Recently G. C. Schmidt,[1] who has examined the case of cadmium iodide in detail, has advanced the view that these successive actions take place in the vapor of the salt. It does not seem to the writer that the facts can be explained on such a simple hypothesis. On such a view it is difficult to see why the ionization should diminish with time when an excess of salt is present and the pressure and temperature are kept constant. For, under these conditions the supply of the CdI_2 vapor is maintained at a steady rate by the salt present. It would seem that in order to explain these effects it is necessary to take into account the existence in the solid salts of unstable forms which have not been detected by chemical methods. This conclusion is supported by the fact observed by Schmidt that the distilled salt is not so efficient as that which has not been distilled.

PALMER LABORATORY,
PRINCETON, N. J.

DISTORTIONS IN SPECTRAL LUMINOSITY CURVES PRODUCED BY VARIATIONS IN THE CHARACTER OF THE COMPARISON STANDARD AND IN THE SURROUNDINGS OF THE PHOTOMETRIC FIELD.[2]

BY HERBERT E. IVES.

THE ordinary procedure in obtaining spectral luminosity curves by either the equality of brightness or the flicker photometer is to compare the spectral colors with a constant standard of arbitrarily chosen color. Thus Koenig compared all the spectrum against wave-length .535μ, Abney used the white light of an arc, the writer has used the light of a standard incandescent lamp. Some previous experiments having led to the suspicion that one's

[1] Ann. der. Physik, Vol. 35, p. 401, 1911.

[2] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

estimation of relative brightness of a set of colors such as the spectrum is altered by a change in the color or character of the comparison standard, a series of experiments have been carried out to clear up this point.

For the incandescent lamp light used in the earlier experiments of this series[1] were substituted a red light, and a green light, chosen so as to give the same intensity of illumination as the "white light." A large number of luminosity curves were then obtained by both the equality of brightness and flicker methods for a high and a low illumination and for large and small photometric fields, using the three colors of comparison field. These showed:

1. By the equality of brightness method the luminosity curves are subject to large shifts and distortions depending upon the color of the comparison field. These shifts are not consistent either in direction or amount in repetitions of the experiment, appearing therefore to consist in disturbance of the observer's judgment, or criterion of equal brightness. By adhering to one comparison light for a long period it has been found that a fairly constant criterion is formed, and consistent results are obtained which might lead one to believe the criterion a correct one. This common belief is however untenable upon considering the family of different curves shown by these experiments to be obtained by the use of several different comparison sources. Particularly disturbing to this belief is the fact that no one of those curves may be depended upon to be reproducible after the observer's criterion is upset by work with another comparison source and he has formed a criterion of brightness with it. The equality of brightness method luminosity curves of Koenig and Abney may easily be considerably shifted from what we may designate as "normal" (to be determined by some other method) due to this cause.

2. Using the flicker photometer and making the same changes in the color of the comparison standard no variations were found in the luminosity curves larger than the errors of measurement.

Following these experiments similar ones were performed with a bright field surrounding the photometer field, in place of the black space usual in optical instruments. Illuminations, field size, and color of comparison source were varied as before. These showed:

1. By the equality of brightness method additional distortions similar to those produced by changing the comparison field color result. Again there appeared to be no regularity or constancy to these shifts. Varying the brightness of the surrounding field hence acts also merely as a judgment-disturbing factor.

2. With the flicker photometer no variations were found in the luminosity curves, greater than the errors of measurement.

3. With a small field, bright surroundings considerably increase the sensibility of equality of brightness settings.

4. The Purkinje effect (equality of brightness method) and the reverse of that effect (flicker photometer) are dependent in magnitude only on the size

[1] PHYS. REV., XXIX., 4, p. 441; PHYS. REV., XXXII., 6, p. 561.

of the photometric field, and not on the total area of illuminated field, in the case where a bright surrounding field is used.

The method of obtaining spectral luminosity curves by steps of small hue difference was investigated. Starting at either end of the spectrum the brightness of adjacent portions was measured, passing by small wave-length differences to the other end. This method was found extremely susceptible to cumulative errors and to errors caused by favoring almost imperceptible differences in the two portions of the photometric field. In order to avoid all possibility of these errors the procedure was adopted of measuring the ratio $\lambda/(\lambda - \Delta\lambda)$ haphazard at numerous points in the spectrum, each measurement being performed so that a complete reversal of all parts took place. For $\Delta\lambda$ was chosen $.004\mu$. From the plot of these ratios a luminosity curve was built up, which, by its method of determination, minimized the possibility of disturbances such as those above considered. At the same time a flicker luminosity curve was obtained. These showed:

At high illuminations (30 m.c.) for a small field (size of fovea) the spectral luminosity curve derived by measurements of small steps of negligible hue difference agrees within the errors of measurement, with the curve obtained by the flicker photometer.

The conclusion drawn from this work is that the flicker photometer, under the conditions named, eliminates all disturbing subjective factors and indicates the true brightness of different colors. This confirms the conclusion drawn as extremely probable from the previous work with five different observers, whose mean equality of brightness curve agreed with their mean flicker curve, at high illuminations.

The Addition of Luminosities of Different Color.[1]

By Herbert E. Ives.

An indispensable characteristic of a method of measurement is that the sum of the measured values of the parts shall equal the measured value of the whole. In order for this to be so it is necessary that things measured equal to the same thing shall measure equal to each other. But this latter property is not alone sufficient, for the process of physical summation may introduce qualities not present in a manner to be measurable in the parts. If these properties are possessed then the ordinary arithmetical processes of addition, subtraction, multiplication and division may be confidently performed with the results of the measurements. In the measurement of most physical quantities and the ordinary photometry of lights of the same color no question of the validity of the measurements arises upon this score. The result of adding two illuminations may be obtained by adding their separately measured values. But in the photometry of lights of different colors, in view of the uncertainties of estimation, and in view of the different results given by

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

different methods it is not permissible to assume that luminosities may be dealt with as ordinary physical quantities.

Spectral luminosity curves have been obtained during the course of the writer's work on heterochromatic photometry, by four different methods, viz., equality of brightness, flicker, critical frequency, and visual acuity. A set of such curves simultaneously obtained, in which the reference standard was the light of an incandescent lamp, showed considerable differences in area. The visual acuity curve was five times the mean area of the others. They in turn differed twenty to thirty per cent. from each other. The large area of the visual acuity curves is due to the chromatic aberration of the eye, owing to which a monochromatic light is far more efficient for defining detail than is a complex light such as the "white" reference standard. Because of this large *quality factor* visual acuity is useless as a method of light measurement. The differences in the areas of the other curves are due to more obscure psychological causes.

Previous work having shown that the two most sensitive methods—equality of brightness and flicker—gave identical results when psychological elements are eliminated, the important test, from the present standpoint, was whether in these methods the arithmetical properties hold. Under conditions where the psychological elements are absent it has been found that brightnesses measured equal to the same measure equal to each other.

There remains to be tested the summational quality. Is there in these photometric methods any quality factor such as chromatic aberration in the case of visual acuity? The most satisfactory test consists in the measurement of the dispersed light of a source color by color against the undispersed light. The sum of the brightnesses of the colors should equal the brightness of the undispersed light. Such an experiment was made by Tufts some years ago, but the conditions of illumination, field size, etc., were far from those indicated as best by the writer's work.

A special slit on the spectrometer made it possible to obtain successively adjacent portions of the spectrum to be measured against the "white" comparison standard, by the flicker method. At the end of the measurements, the slit was opened until the whole spectrum fell upon the eye-slit, when both sides of the photometric field appeared of the same color, and a photometric match could be obtained by all methods of photometry with necessarily identical results. The experiment was carried out at an illumination of 300 illumination units, photometric field 2° diameter, with bright surroundings.

The result proved the physical and arithmetical summations identical.

It follows from this and the previous work reported before the Physical Society that in the flicker photometer we possess an instrument which under proper conditions offers a practical solution of the problem of heterochromatic photometry. There remains only to be determined the luminosity curve of the normal eye, by measurements on numerous individuals, in order that standard conditions for the photometry of different colored lights may be specified and made the subject of legislation.

With the completion of this work will be rendered possible the establishment of a logical primary standard of light, namely, a definite intensity of radiation of maximum luminous efficiency, in accordance with the proposals of Ives, Strache and Houstoun.

Accurate Potentiometer Method for Measuring Resistance.[1]

By Walter P. White.

IN this method, the resistance is made part of a Wheatstone's bridge which is not changed during the observations. The lack of balance, which may be relatively large, is not allowed to send a current through a galvanometer, but is measured by putting a potentiometer in the place usually occupied by the galvanometer alone. The method is best suited for measuring resistance changes rather than absolute values. It is obviously more accurate than the ordinary potentiometer methods, since the electrical quantity to be measured is much smaller, and changes in the battery current produce far less effect. Its advantage over the simple Wheatstone bridge lies in the fact that no troublesome contact resistance changes occur during the measurements, and that the general advantages of the potentiometer can be secured along with some of the benefits of the Wheatstone bridge.

It does not avoid the difficulty from temperature change in the coils which occurs in Wheatstone bridge work of high precision, but it facilitates the use of some methods which diminish this difficulty also. For instance, in temperature measurement it facilitates the use of resistance thermometers with two (or four) coils in the bulb, half of them of wire of low temperature coefficient. The suggestion may be worth while that some of the disadvantages which appear to have prevented the use of thermometers of this type might be obviated if the metal of low temperature coefficient should be the alloy of silver and platinum, which would endure much the same heat treatment as the pure platinum and very much more than would be admissible with manganin.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C.,
April 23, 1912.

On Secondary Ionization Produced by the Impact of Positive Ions on Solids.[1]

By O. W. Richardson and Charles Sheard.

THE incentive to the present investigation was an observation recorded by one of the authors[2] to the effect that when a fresh metal wire is positively charged and heated in a vacuum the relation between current and electro-

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

[2] O. W. Richardson, Phil. Mag., Vol. 6, p. 80, 1903.

motive force is approximately linear, although there are no negative ions present. Since one would expect saturation to occur with a quite small potential under these circumstances this phenomenon seemed to call for further examination. The experiments have been made with three forms of testing vessels, (1) a glass tube in which the electrodes were a loop of heated platinum wire and a platinum plate, (2) a brass cylinder with an axial platinum wire, (3) a brass cylinder with two parallel platinum wires which could be heated and tested separately. All the currents were very small and were measured by a delicate electrometer. The pressure recorded on the McLeod gauge was between the limits 0.00005 and 0.0002 mm. in all the experiments.

In every case when the apparatus was newly set up the relation between current and potential was approximately linear from +40 to +400 volts, although as a rule there was a slight convexity towards the voltage axis. This relation did not hold below 40 volts; saturation usually occurred below 5 volts and in some instances the current with 3 volts was distinctly greater than with 30 volts. The increase in the currents above 40 volts is evidently due to some new source of ionization being brought into play by the field and it is much too large to be attributable to ionization by collisions with the molecules of the residual gas.

The large additional current at high voltages was found to gradually disappear with continued usage of the tube if the platinum wire was kept heated and charged positively. It was also cut down very considerably if the tube was placed in a magnetic field. These effects were not observed if the negative emission from the heated wire was tested in a similar manner.

When the large additional current at high voltages had disappeared with usage of the tube it could be restored by the following processes:

1. Heating the positively charged wire to a higher temperature than that at which it had been tested previously.

2. Allowing a current of negative electrons to pass from the wire to the other electrode.

3. Allowing the apparatus to stand with air at atmospheric pressure in it. The effect of letting air in was found to give similar results whether the tube was scraped inside or not and it could not be removed by continued pumping unless a discharge of positive ions was allowed to pass to the electrode at the same time.

The results can all be explained on the hypothesis that the various agencies 1–3 deposit a layer of material on the opposite electrode which readily emits electrons when bombarded by the positive ions. The effect of the magnetic field shows that the secondary ions thus emitted are electrons and this conclusion is confirmed by the fact that similar effects are not observed when the primary discharge is negative instead of positive.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

The Photoelectric Effect.[1]

By O. W. Richardson and Karl T. Compton.

THE writers have recently been engaged in an investigation of the magnitude and distribution of the total kinetic energy of the electrons emitted when light falls on metals, considered as a function of the frequency of the light and of the nature of the metal.

The light from a Heræus quartz mercury arc lamp after passing through a Hilger monochromatic ultra-violet illuminator fell on a small narrow strip of the metal to be investigated. This was placed at the center of an exhausted glass bulb provided with a quartz window. The inside of the bulb was silvered and was connected to the insulated quadrants of a delicate electrometer. The current which flowed into the electrometer against various opposing potentials was measured. As the field is everywhere radial these currents give the magnitude and distribution of the kinetic energy of the emitted electrons directly. The correction for the reflected electrons has not yet been determined, but is probably small with this arrangement, as the chance of a reflected electron being returned to the central electrode is small.

The experimental results may be analyzed and exhibited graphically by plotting the number of electrons having a given energy against the energy. These curves differ somewhat in form in different cases but they are nearly symmetrical about the axis of mean energy. The number also is a maximum near this value, so that the mean energy is the most probable value of the energy. Equal deviations from the mean have also nearly equal probabilities. The probability of an electron having an amount of energy between given limits changes very rapidly in the neighborhood of the maximum energy and also in the neighborhood of zero energy. The range of the energy of the electrons emitted is approximately a linear function of the frequency; *i. e.*, the maximum energy is a linear function of the frequency.

In a paper read before the meeting of the Society on March 2, 1912, one of the writers gave reasons for believing that the average kinetic energy $\overline{T_\nu}$ of the electrons emitted by the action of light of frequency ν might be expected to be capable of representation for all substances by a relation of the form

$$\overline{T_\nu} = \nu F\left(\frac{w_p}{\nu}\right), \qquad (1)$$

where F denotes a universal function whose form depends on the energy distribution in the black-body spectrum and w_p is the energy required to set free a negative electron from the material denoted by the suffix p. The equation (1) is derived from the solution of two integral equations, one between equivalent expressions for the number of electrons emitted in a given time and the other between equivalent expressions for the energy lost in a given time. Ac-

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

cording to equation (1) the mean energy of the electrons emitted by light of frequency v when it falls on different bodies is determined entirely by v and a single parameter w_p characteristic of the body. Thus if the graph of $\overline{T}_v$ against v were known for any one substance it could at once be predicted for any other, provided the appropriate value of w could be determined. Provided the electrons which participate in the photo-electric effect are in the same class as those which give rise to the thermionic emission the w's are closely connected with the intrinsic potentials of the substances, the relation between them being, in fact, if the suffixes m and p refer to two different materials,

$$\frac{w_m}{e} - \frac{w_p}{e} = V_m - \theta \frac{\partial V_m}{\partial \theta} - \left(V_p - \theta \frac{\partial V_p}{\partial \theta} \right). \tag{2}$$

If, for the present, we disregard the terms depending on $\partial V/\partial\theta$ as small (this is legitimate since $(\partial/\partial\theta)(V_m - V_p)$ is equal to the thermo-electric power of a circuit consisting of the two metals) then the differences of w/e are equal to the contact differences of potential. Moreover the absolute value of w may be obtained, although perhaps only approximately at present, from the results of experiments on the variation with temperature of the thermionic emission and on the heat developed when electrons are made to condense into a metal. Such experiments show that the value of w/e for platinum is not far from 5 volts. We have therefore independent methods of obtaining both the absolute and relative values of w/e for different substances.

It can be shown that (1) may be written in the equivalent forms

$$\overline{T}_v = v\varphi_1 \left(\frac{v_p}{v} \right) \tag{3}$$

and

$$\lambda \overline{T}_\lambda = \varphi_2 \left(\frac{\lambda}{\lambda_p} \right), \tag{4}$$

where φ_1 and φ_2 are universal functions, λ is wave-length and $v_p = c/\lambda_p$ is a frequency which is proportional to w_p. According to Planck's radiation formula $w_p = hv_p$ where h is Planck's constant.

The experimental results have been compared with (4) by plotting $\lambda\overline{T}_\lambda$ as ordinates against λ/λ_p as abscissæ. The first metal tested showed that when $\lambda\overline{T}_\lambda$ was plotted against λ the curve was a straight line which intersected the axis of λ at a certain point. Calling this point λ_p the equation to the curve when plotted against λ/λ_p thus becomes $\lambda\overline{T}_\lambda = k(1 - \lambda/\lambda_p)$. The values of λ_p for the other substances were determined by making one of the experimental points fall on this line. When this was done all the other points were found to lie on the line in the case of the following metals: sodium, aluminium, magnesium, zinc, tin and platinum. In the case of sodium, however, we were not able to determine the average energy directly on account of photoelectric fatigue. The average was estimated from determinations of the maximum energy of the particles in this case. This procedure, however, gave pretty good

values when applied to the other substances. The behavior of bismuth and copper, which gave very small currents, appeared to be exceptional. The points for these two metals lay on a line through the origin having a different slope. The constant k for the metals Na, Al, Mg, Zn, Sn and Pt was $C \times 2.9 \times 10^{-27}$ whereas for Bi and Cu it appeared to be about $C \times 1.9 \times 10^{-27}$, where $C = 3 \times 10^{10}$ cm. per sec. The value of Planck's constant h is 6.55×10^{-27} in the same units (1 = 1 erg $\times$ 1 sec.).

It will be seen that the values of λ_p which were used were obtained entirely from the photo-electric measurements. Assuming the truth of Planck's radiation formula values of λ_p may be calculated from the radiation constant h, the absolute value of w for any one metal and the contact differences of potential. Taking $w/e = 5$ volts for platinum the calculated values of λ_p are given in the first row of the following table and those from the photo-electric measurements in the second row.

Metal.	Na	Al	Mg	Zn	Sn	Bi	Cu	Pt
λ_p (calculated from w)	52.6	36.0	34.6	33.3	31.0	29.4	28.0	27.3
λ_p (photo-electric)	57.6	39.5	36.5	36.1	33.8	33.1	29.7	29.0

These numbers have been obtained quite independently, and no attempt has been made to adjust the data, so that the agreement shown in the table is really quite good. Values of λ_p were also obtained by trying to find the longest waves which would give rise to any photoelectric emission. These agreed with those given in the table. The agreement in the case of bismuth and copper is not in itself an argument either for or against the theory as the points for these substances did not fall on the general curve.

The values which we have found for the maximum energy of the emitted electrons T_{max} have also been analyzed by plotting λT_{max} against λ/λ_p. Although the points are a little irregular they all fall fairly near a line cutting the axis of abscissæ at $\lambda/\lambda_p = 1$, whose equation is $\lambda T_\lambda = ch(1 - \lambda/\lambda_p)$, where c is the velocity of light and h is Planck's constant. They could be better represented by a linear equation with a smaller value of h than that given by the radiation formula (about 10 per cent. lower). We do not, however, wish to emphasize this difference, pending further investigation, as we realize that the accurate measurement of the maximum energy is a rather difficult problem. So far as they go, the measurements of the maximum energy may be considered favorable to Einstein's[1] theory of the photo-electric effect combined with the hypotheiss that the difference in the work P for different substances is determined by the contact difference of potential.

If the laws which we have found to connect the frequency of the light with the maximum and mean energy of the liberated electrons hold up to the highest frequencies, it follows that the frequency v of homogeneous Roentgen rays can be obtained from either of the equations

[1] Ann. der Physik., Vol. 17, p. 146, 1905.

$$v - v_p = \frac{\overline{T}}{2.9} \times 10^{27} = \frac{T_{max}}{6} \times 10^{27},$$

where $\overline{T}$ is the mean energy, and T_{max} the maximum energy, of the electrons emitted when the Roentgen rays fall on a metal. For these high frequencies v_p can be neglected compared with v. Sadler[1] and Beattie[2] have shown that the energy of the electrons emitted when Roentgen rays fall on different substances is determined solely by the character of the Roentgen rays and is independent of the nature of the substance. This is in accordance with the requirements of the above formulæ.

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HEATS OF DILUTION.[3]

BY WILLIAM FRANCIS MAGIE.

WHEN an aqueous solution containing a gram-molecule of an electrolyte is diluted by the addition of unit volume of the solvent the heat evolved (reckoned positive) or absorbed (reckoned negative) is the heat of dilution. It diminishes as the volume of the solution increases and changes remarkably with the temperature. A well-known thermodynamic argument shows that the rate of change of the heat of dilution with the temperature equals the negative rate of change of the heat capacity of the solution and the solvent with the volume of the solution, as it is increased by the transfer of solvent to the solution.

This theorem has been verified as closely as the simple method of observation used would permit, for solutions of the chlorides of sodium, potassium, ammonium, barium and strontium, within the range of concentrations from 4 to 0.5. The concentrations are expressed in an arbitrary unit by $100/N$, where N is the number of gram-molecules of water containing one gram-molecule of the solute. For the last three substances the heats of dilution are generally negative at the lower temperatures used (about 5° C.) and positive at the higher temperatures (about 24° C.). Observations have been made at temperatures indicated from these observations as the transition temperatures (at which there is no heat of dilution) for certain concentrations and the indications of the theory confirmed.

From the fact that the formula connecting the temperature change of the heat of dilution with the volume change of the heat capacity is verified for finite ranges of temperature, as well as from the direct observations of Teudt, it appears that the change of heat capacity with the volume is independent of temperature. This being assumed we have

[1] Nature, Vol. 81, p. 516, 1909; Phil. Mag., Vol. 19, p. 337, 1910.

[2] Camb. Phil. Proc., Vol. 15, p. 416, 1910.

[3] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

$$l = -a\theta + e,$$

in which l is the heat of dilution, a, the rate of change of heat capacity with the volume, θ, the absolute temperature, and e, a function of the volume but not of the temperature. The factor a is negative and proportional to the rate of change of the dissociation, measured by the electrolytic conductivity, with the volume. The first term is positive and corresponds to an evolution of heat. It may be explained, on the theory previously supported by the author, by supposing that when dissociation takes place, the water released from the molecules which dissociate has its heat capacity diminished, and the water associated with the newly formed ions has its heat capacity also diminished, so that the number of degrees of freedom of the water is diminished for both reasons, and the energy associated with the degrees of freedom which are thus removed from the solution is released in the solution in the form of heat. This energy, at these temperatures, is proportional to the absolute temperature. The term e is negative and corresponds to an absorption of heat. It is to a first approximation proportional to the rate of change of dissociation with the volume. When reduced to complete dissociation the heat absorbed is of the same order of magnitude as the heat of combination of the elements composing the solute. The term e may therefore contain, as a principal part of it, the heat of dissociation.

The Relation of Osmotic Pressure to Temperature.[1]

By William Francis Magie.

If we represent by p the osmotic pressure, by θ the absolute temperature, and by a the rate of change of heat capacity of an aqueous solution of an electrolyte with the volume (see the previous paper) we may write the following relation

$$\frac{d^2p}{d\theta^2} = \frac{a}{\theta}.$$

Assuming that a is independent of the temperature we obtain by integration

$$p = a\theta(\log\theta - 1) + b\theta + e,$$

in which a, b, e are functions of the volume but not of the temperature.

By a simple thermodynamic argument we can deduce from this the following expression for the heat of dilution,

$$l = -a\theta + e.$$

Now a is known from observations on the specific heats of solutions, e can therefore be determined from the observed heats of dilution, and b from values of the osmotic pressure calculated from observations of the depression of the freezing point. The formula can then be tested by calculating with it the osmotic pressure at the boiling point and comparing the result with the osmotic

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

pressure calculated from observations of the elevation of the boiling point. The only data at present available are those for sodium chloride. The observations of Thomsen furnish the heat capacities, those of the author, the heats of dilution, those of Kahlenberg, the freezing and boiling points. This example confirms the formula very precisely for solutions containing one gram-molecule of salt to 50, 100, and 200 gram-molecules of water.

Furthermore we can calculate from the formula for osmotic pressure, when the constants are known, the ratio of the vapor pressure over the pure solvent to that over a solution of a given strength at different temperatures. According to von Babo's law this ratio should be constant. From the theoretical formula for this ratio, its constancy at different temperatures is immediately evident if we assume, as has commonly been done, that the osmotic pressure is directly proportional to the absolute temperature. With the more complicated relation of osmotic pressure to temperature expressed in the proposed formula, von Babo's law can only be tested by direct calculation for each solute. In the case of sodium chloride, for which the constants of the formula have been obtained, it turns out that the law is verified, the departures of the ratios for different temperatures between 0° and 100° C. from the mean being never so great as those which would arise from errors of observation.

The form of the formula for osmotic pressure is the same as that for the negative rate of change of the free energy with the volume (equal to the pressure) of a gas which conforms to van der Waals' equation.

The constants of the formula evidently depend upon the interactions between the parts of the solute and the solvent. The formula for the osmotic pressure is consistent with the view that the osmotic pressure is not a purely kinetic pressure, but is due to forces acting between the parts of the solute and the solvent.

THE LAW OF FALL OF A DROP THROUGH AIR AT REDUCED PRESSURES AND A REDETERMINATION OF e.[1]

BY R. A. MILLIKAN.

IN work previously reported[2] it was shown for the first time that Stokes' Law of fall was insufficient to account for the motions of small drops through air and a corrected law of the form

$$X = 6\pi\mu av\left(1 + A\frac{l}{a}\right)^{-1} \qquad (1)$$

was proposed and shown to hold in air at atmospheric pressure for large variations in a. By observing the motions of oil drops in gravitational and electrical fields, this law has now been tested for pressures between 5 mm. and 760 mm. and found to hold accurately so long as $l/a < .4$, *i. e.*, so long as

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

[2] The Isolation of an Ion, etc., PHYS. REV., Vol. XXXII., pp. 349–397.

the radius of the drop is more than twice the mean free path of the gas molecule. When $l/a > .4$ the above equation fails and must be replaced by one containing higher powers of l/a. The value of A in (1) has now been found with a much greater precision than had before been possible. It is found to be .848 instead of .817, the value previously reported. This coupled with a redetermination of all of the factors involved in e has made it possible to obtain a value of this constant the probable error of which is about .1 per cent. The new value of e is 4.810×10^{-10}.

Variation of Electrical Resistance with Temperature. V. — Oxides.[1]

By A. A. Somerville.

A CONTINUATION of work the beginning of which was reported at the New York meeting of the American Physical Society, held March 2, 1912. The resistance of oxides is determined by placing the powder in a porcelain tube, inserting nickel terminals into the tube, putting the whole thing into a tubular resistance furnace and measuring the resistance between the nickel terminals with a wheatstone bridge, when the furnace is at any temperature from room temperature to 1100° C.

The materials that have been studied are—

Zinc oxide	ZnO
Ferric oxide	Fe_2O_3
Cupric oxide	CuO
Cuprous oxide	Cu_2O
Magnesium oxide	MgO
Manganese dioxide	MnO_2
Aluminum oxide	Al_2O_3

All are insulators under ordinary conditions, but the resistance decreases as temperature increases until most of them at 1100° C. are fairly good electrical conductors. The general form of the temperature-resistance curves is the same, being something like that of a parabola, but the actual value of the resistance of various materials at the same temperature differs widely.

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[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

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